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Mathematical modeling and analytical solution for stretching force of automatic feed mechanism

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Abstract: Load of an automatic feed mechanism is composed of the stretching force of feed belt at the entrance to lower flexible guidance and the friction force between feed belt and flexible guidance. A mathematical model for computing the load was presented. An optimization problem was formulated to determine the attitude of the flexible guidance based on the principle that the potential energy stored in the system was the minimum at the equilibrium. Then the friction force was obtained according to the attitude of guide leaves and the moving velocity of the feed belt and the friction factor. Consequently, the load of the automatic feed mechanism can be calculated. Finally, an example was given to compute the load when the horizontal and elevating firing angles of the automation were respectively 45˚ and 30˚. The computing result can be a criterion to determine the designing parameters of automat.

Key words: mathematical model; load; automatic feed mechanism; optimization problem; potential energy; hoist-feed system

1 Introduction

In order to assure automatic and continuous ability to long or short burst of minor shipboard armament with bigger basic ammunition in many complicated conditions, the feeding system must have adaptive ability. So, the study on the adaptive technique of ammunition hoisting and feeding of minor shipboard armament is more and more important. Recently, researchers have studied the modeling and simulation of the feeding system of naval gun[1−2] and the projectile queue motion of magazine system[3−4].

Self-adaptive motion of hoisting and feeding of naval gun is a complex motion that projectiles timely transfer from one position (the initial position) to the other (the terminal position) at the set time. Generally, the trajectory of projectiles is a space curve that varies with elevating and rotating of the gun. And guidance technology is applied to ensuring that ammunition motion trajectory must be adapted for continuously varying of position[5], where the automatic feed mechanism receives ammunitions. Therefore, the channels composed of flexible guidance must be adapted for varying of receiving position of automat. Evidently, it is important to study the problem computing the load of the automat, and it is key to determine the relation of flexible guidance attitude with elevation and rotation of gun.

By analyzing the structure and movement characteristics of flexible guidance, and according to the theory that potential energy of the flexible guiding system in natural equilibrium state was the minimum, an optimization model for solving three-dimensional attitude of flexible guidance was proposed. Also, a mathematical model for computing the load was presented. Then the corresponding calculating software was developed to determine the attitude of the flexible guidance and the load of the automat[6−7]. It is of great value to study the interaction between feed belt and flexible guidance, and to determine design parameters of automat.

A completely different technique would be the application of the minimum potential energy principle perse: ‘of all the displacements which satisfy the boundary condition of a structural system, those corresponding to the stable equilibrium configurations make the potential energy a relative minimum’[8], Theorem X. This principle, although is well known and applied through illustrative examples in almost every structural analysis book, is not thoroughly exploited except for very few cases[9−10]. In the present work, it is shown that its application to solving the attitude of the flexible structural is very fruitful. The problem, formulated as an application of the minimum potential energy principle, is obviously an optimization problem. So, the penalty method with slack variables is used for solving the problem.

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2 Specification of feeding process

Feeding process is defined as the moving process of navel shells with belt in the channel composed of upper and lower flexible guide. In the process, the flowing direction of projectile trace changes with channel, and its flowing drive force comes from recoil energy of naval gun. The top end of upper guidance is fixed with the automat, the bottom end of lower guidance is connected with the fixed caisson through the straight guidance and helical guidance, and upper and lower guidance are joined by transition part. Because the automat rotates and elevates along with rotating and elevating part of the naval gun, respectively, and the caisson is installed on the fixed part of the naval gun. The joint of upper guidance and the automat is the moving point while the joint of lower guidance and caisson is the fixed point. It is such a simple and smart structure that naval shells are ensured to arrive at the correct position where the automat receives them, as long as they enter into the channel. To keep continuity of the channel, guidance is designed as the structure flexibly joining many guide leaves together, in which each guide leaf has 4 DOFs relative to neighboring guide leaf[11]. So, the guiding system is called flexible guidance, and continuity and accuracy of channel are ensured, where the deformation of the upper and the lower guidance is independent of one another. The upper flexible guidance mainly adjusts with elevating of naval gun while the lower flexible guidance mainly adjusts with rotating of naval gun. In other words, guide leaves of the lower guidance relatively rotate and move with horizontally rotating of gun while the attitude of the upper guidance is unaltered; similarly, the attitude of the upper guidance varies with elevating of gun while attitude of lower guidance is unaltered. So, the attitudes of the upper and lower guidance can be separately solved. In fact, as long as the attitude and position of each guide leaf are known at any fire angle and direction, the attitude of flexible guidance system is determined.

At last, the projectiles from feed mechanism are transferred to the feed route by the driving force from the automat.

3 Problem formulations

The ammunition feed mechanism is an important part of a roll-tube and high-firing frequency naval gun. It can provide force to feed projectiles for firing. In fact, the load is the stretching force of the feed belt at the entrance to the automat. And it is composed of two parts: the stretching force of the feed belt at the entrance to lower flexible guidance and the friction force between the feed belt and the flexible guidance. The former may be calculated by the linear moving velocity and density of the feed belt according to the momentum law, while the latter can be determined by the central position of each guide leaf and the linear moving velocity of the feed belt and the friction factor and so on.

3.1 Stretching force of feed belt

F_0 is expressed as the initial stretching force to draw the feed belt from the fixed caisson, F_f as the sum of the friction forces arising from several crooks of the fixed guidance, and F_1 as the stretching force of the feed belt at the entrance to the lower flexible guidance. The following relation can be written as:

$$F_1=F_0+F_f \tag{1}$$

The following relations can be deduced according to the momentum law, where v and ρ are expressed as the linear velocity and density of the feed felt, respectively.

$$F_0\Delta t=(\Delta tv\rho) \tag{2}$$

So,

$$F_0=\rho v^2 \tag{3}$$

If F_f is ignored, the following expressions can be obtained:

$$F_1=F_0 \tag{4}$$

3.2 Friction force between feed belt and flexible guidance

Providing that r is defined as the radius of curvature at any place of the flexible guidance, the normal pressure applied by the feed belt to the flexible guidance is written as follows:

$$F_n=mv^2/r \tag{5}$$

F_2 is defined as the friction force between the feed belt and the flexible guidance, so,

$$F_2 = f\sum_{i=1}^{n-1} nv^2/r_i \tag{6}$$

where n is the number of all nonlinear subsections of the guidance, f is the friction factor, and r_i is the radius of the circle formed by three points that are the central positions of three adjacent guide leaves, respectively.

3.3 Load of automat

Summarizing, the load of the automat is

$$F=F_1+F_2 \tag{7}$$

As long as the loads are respectively calculated at any horizontal and elevating angles of fire, the total load of the automat can be determined by the sum of the both at the corresponding horizontal and elevating angles.

3.4 Key of problem

It is key to determine the central position of all guide leaves in any fire direction and angles of naval gun.

4 Modeling of central position of guide leaves

4.1 Basic assumption

Cartridge belt and chain are regarded as regular linear flexible body, and flexible guidance is regarded as the elastic system composed of multi-rigid body, where guide leaf of link belt is a rigid body and adjacent guide leaves are connected each other by elastic component.

4.2 Kinematic analysis of feed transfer flexible guidance

The relative movement between the adjacent guide leaves consists of three rotations around *x*-axis, *y*-axis and *z*-axis and one parallel movement along *z*-axis. No.*i* and No.*i*−1 guide leaves are expressed by B_i and B_{i-1}, respectively. Firstly, B_i rotates θ_i around *y*-axis of B_{i-1} coordinate system, and rotates ψ_i around *x*-axis of B_{i-1} coordinate system, then rotates φ_i around *z*-axis of B_{i-1} coordinate system, lastly parallelly moves Δz_i along *z*-axis of B_{i-1} coordinate system. Abridged general view of initial position of B_i relative to B_{i-1} is shown in Fig.1. Abridged general parallel movement view of B_i relative to B_{i-1} along *z*-axis is shown in Fig.2. Abridged general rotation view of B_i relative to B_{i-1} around *y*-axis is shown in Fig.3. Abridged general rotation view of B_i relative to B_{i-1} around *x*-axis is shown in Fig.4, and abridged general rotation view of B_i relative to B_{i-1} around *z*-axis is shown in Fig.5. So, the attitude of flexible guidance is described by three rotation angles θ_i, ψ_i, φ_i (i=1, 2, ⋯, n−1) and one parallel displacement Δz_i (i=1, 2, ⋯, n−1).

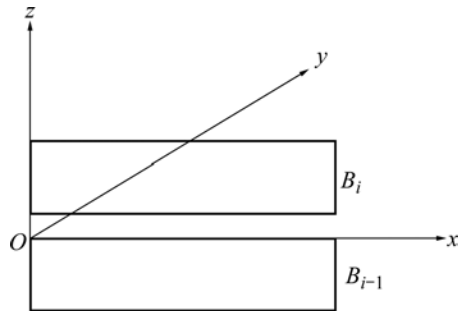

Fig.1 Initial position of B_i relative to B_{i-1}

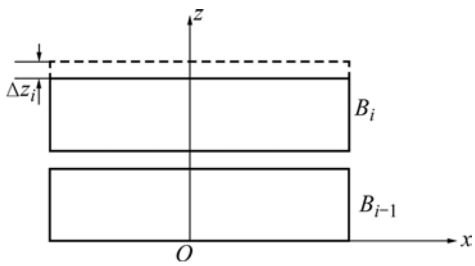

Fig.2 B_i translating Δz_i along *z* axis based on B_{i-1} (*z*-axis points outwards vertically)

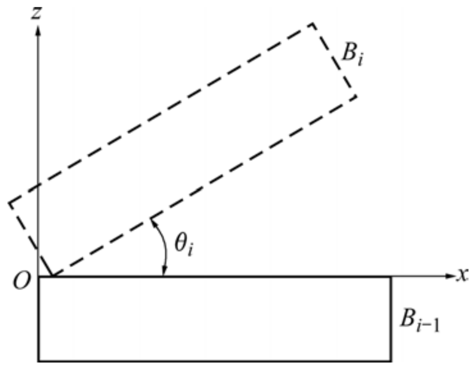

Fig.3 B_i rotating θ_i round *y*-axis based on B_{i-1} (*y* axis points inside vertically)

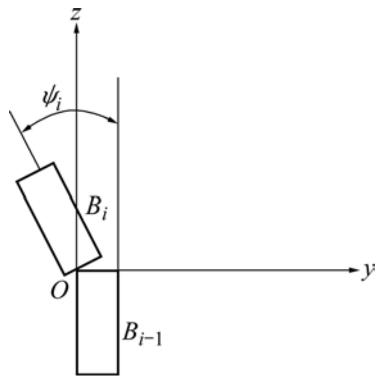

Fig.4 B_i rotating ψ_i round *x*-axis based on B_{i-1} (*x*-axis points outward vertically)

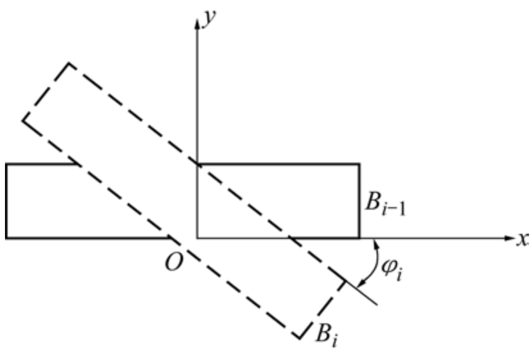

Fig.5 B_i rotating φ_i round *z*-axis based on B_{i-1} (*z* axis points outward vertically)

4.3 Restriction equations

n is the number of guide leaves of the upper or the lower flexible guidance, the serial number of guide leaf counts from 0, and B_0 coordinate system is a fixed coordinate system. Kinematics relation of flexible guidance is deduced as follows.

4.3.1 Attitude of guidance

$\boldsymbol{A}_j$ is defined as the coordinate transition matrix from the guide leaf B_{i-1} to B_i, which is the function of θ_j, φ_j, and ψ_j.

$$\boldsymbol{A}_j=\begin{bmatrix}\cos\theta_j & 0 & -\sin\theta_j\\ 0 & 1 & 0\\ \sin\theta_j & 0 & \cos\theta_j\end{bmatrix}\cdot\begin{bmatrix}\cos\psi_j & \sin\psi_j & 0\\ -\sin\psi_j & \cos\psi_j & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi_j & \sin\varphi_j\\ 0 & -\sin\varphi_j & \cos\varphi_j\end{bmatrix} \quad (8)$$

$$\prod_{j=1}^{n}\boldsymbol{A}_j=\boldsymbol{A}_{0n} \quad (9)$$

where $\boldsymbol{A}_{0n}$ is the coordinate transition matrix from the guide leaf B_0 to B_{0-1} and is determined by the attitudes of the guide leaf B_0 and B_{n-1} which are known.

4.3.2 Center position of guide leaves

The slippage vector $\Delta\boldsymbol{z}_i$ is fixed with the rigid body B_i, and $\Delta\boldsymbol{z}_i=[0, 0, l+\Delta z_i]^{\mathrm{T}}$. So,

$$\boldsymbol{r}_i=\boldsymbol{r}_{i-1}+\Delta\boldsymbol{z}_i \quad (10)$$

$$r_i=r_{i-1}+\boldsymbol{W}_i^{\mathrm{T}}\Delta z_i \quad (11)$$

$$r_{n-1}=r_0+\sum_{i=1}^{n-1}\boldsymbol{W}_i^{\mathrm{T}}\Delta z_i \quad (12)$$

$$\sum_{i=1}^{n-1}\boldsymbol{W}_i^{\mathrm{T}}\Delta z_i=r_{n-1}-r_0 \quad (13)$$

where l is the length of guide leaf, and $\boldsymbol{r}_i$ is the vector from the fixed coordinate origin to the center of guide leaf B_i, and $\boldsymbol{W}_i^{\mathrm{T}}$ is the coordinate transition matrix from guide leaf B_i to B_0.

$$\boldsymbol{W}_i=\prod_{j=1}^{i}\boldsymbol{A}_j \quad (14)$$

4.4 Mathematical model

Positions of guide leaves are determined by the positions of the guide leaf B_0 and B_{n-1}, and θ_i, ψ_i, φ_i, and Δz_i (i=1, 2, ⋯, n−1) between adjacent guide leaves. Evidently, the number of equations is far less than the number of variables, so this is a statically indeterminate problem, and it is necessary for solving the natural attitude of flexible guidance to introduce the principle of the minimum potential energy[12−13]. That is to say, the potential energy stored in the guide system in the natural equilibrium state is the minimum[14−15].

4.4.1 Potential energy

Because the guidance is the elastic system composed of multi-rigid body, the potential energy of the system is the sum of potential energy of all the elastic elements between guide leaves.

$$E=E_x+E_y+E_z+E_d=\sum_{i=1}^{n}\frac{1}{2}k_x\psi_i^2+\sum_{i=1}^{n}\frac{1}{2}k_y\theta_i^2+\sum_{i=1}^{n}\frac{1}{2}k_z\varphi_i^2+\sum_{i=1}^{n}\frac{1}{2}k_d\Delta z_i^2 \quad (15)$$

where E is the potential energy of the system; E_x, E_y, and E_z are the potential energy from the rotation round x-axis, y-axis and z-axis, respectively; and E_{d} is the potential energy from the parallel movement along z-axis.

4.4.2 Optimization problem

Hence, the problem is transformed to an optimization problem, where the rotation angle and parallel displacement between adjacent guide leaves of link belt are regarded as variables, the boundary conditions of flexible guidance are as constraints, and the potential energy of the guiding system is objective function. The model is as follows:

$$\min(E=E_x+E_y+E_z+E_{\mathrm{d}})$$

$$\boldsymbol{X}=(\psi_1\cdots\psi_{n-1},\theta_1\cdots\theta_{n-1},\varphi_1\cdots\varphi_{n-1},\Delta z_1\cdots\Delta z_{n-1})^{\mathrm{T}}\in\boldsymbol{D}$$

Subject to:

1) the attitude (the deformation of angle among guide leaves)

$$\prod_{j=1}^{n-1}\boldsymbol{A}_j=\boldsymbol{A}_{0n}$$

2) the position of center of guide leaf

$$\sum_{i=1}^{n-1}\boldsymbol{W}_i^{\mathrm{T}}\Delta z_i=r_{n-1}-r_0$$

5 Computing examples

Because the mathematical model is an optimization model with only one objective function and many equality constraints, penalty-function method can be selected to solve the problem, and BFGS method is applied, which is the widespread and effective method of unrestricted optimization method. Quadratic interpolation is used to solve one-dimensional search problem, and one-dimensional search interval is determined with method of extrapolation, then the corresponding computing software is developed. With the software, the position of each guide leaf can be calculated, and the load of the automat can be determined in any fire direction and angle of naval gun. For example, the attitude of the lower guidance at 45˚ of fire direction was calculated, as shown in Fig.6, and the load is 236.638 8 N. At the same time, the attitude of the upper flexible guidance at 30˚ of fire angle is shown in Fig.7, and the load is 266.259 4 N, so the total load of the automat is 502.898 2 N. The results show that the mathematical model is reasonable.

6 Conclusions

1) The central position of each guide leaf in the flexible guidance can be calculated, and the total load of the automat is consequently determined by the methods

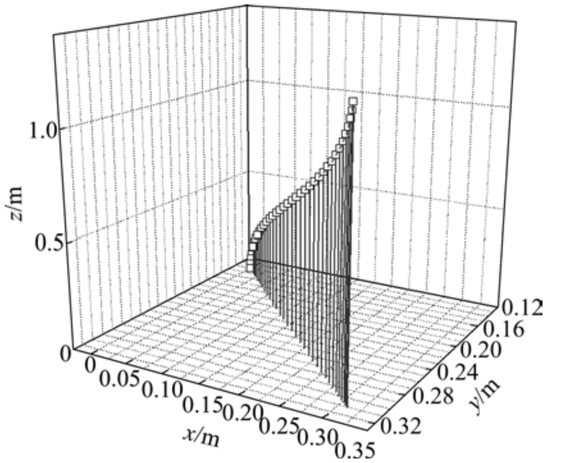

Fig.6 Attitude of lower guidance (horizontal firing direction: 45˚)

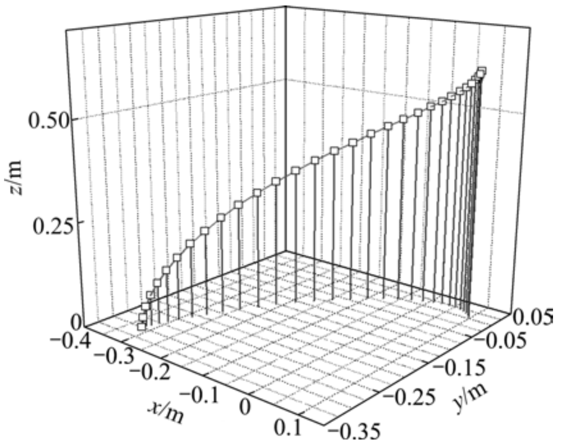

Fig.7 Attitude of upper guidance (elevating angle: 30˚)

developed when the gun's horizontal rotating and elevating angle of fire is any degree.

2) The computation result shows that the mathematical model presented in this work is reasonable, and it can be a criterion to determine the designing parameters of the automat. Moreover, it is significant to study the adaptive technology of the hoist-feed system of the small-bore naval gun.

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