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Design of One Stage Planetary Gear Train With Improved Conditions of Load Distributionand Reduced Transmission Errors

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The authors propose an approach for the design of one-stage planetary gear train with reduced transmission errors, localized bearing contact and improved conditions of distribution of load between the planetary gears. The planetary gear train is considered as a multi-body mechanical system of rigid bodies. The proposed approach is based:

(i) on modification of geometry of gear tooth surfaces, and (ii) minimization and equalization of the backlash between the contacting gear tooth surfaces. The modification of geometry is accomplished: (i) by double-crowning of planetary gears, and (ii) by application of screw
involute surfaces of the same direction of screws for the sun and the ring gears. The proposed geometry enables: (i) predesign of parabolic function of transmission errors for reduction of noise and vibration, and (ii) a simple method of regulation and equalization of the backlash between the gear tooth.

一、Introduction

Planetary gear trains were the subject of intensive research directed at determination of dynamic response of the trains, vibration, load distribution, efficiency, enhanced design and other important topics [1–12]

The contents of this paper cover investigation of one-stage planetary gear train （Fig. 1）. The train is considered as a multibody mechanical system of rigid bodies.

The goals of investigation are: （i） localization of bearing contact, （ii） reduction of transmission errors, and （iii） Minimization and equalization of backlash between the gears of the train.

The regulation of backlash is a simple procedure accomplished in the process of assemble （see section 3） as the precondition for uniform distribution of load between the gears of the train.

The proposed approach developed in this paper enables to improve the meshing and contact of a misaligned one-stage planetary gear drive and is based on the following ideas

1. Instantaneous line contact of tooth surfaces is substituted by point contact. However, the contact under the load is spread over an elliptical area

2. The tooth surfaces of planetary gears are modified and double-crowned, in profile and longitudinal directions. In the existing design, crowning is performed for elimination of the tip of the profile and the edge of tooth surfaces. The approach applied for double-crowning is awarded by a patent

3. Reduction of transmission errors of the train is achieved by application of a predesigned parabolic function of transmission errors. Such function absorbs linear discontinuous functions of transmission errors that is the main source of vibration and noise

4. An approach is developed for minimization and equalization of the backlash between the tooth surfaces of train gears. This is in favor of uniform distribution of load between the gears

The ideas proposed are confirmed by computerized simulation of meshing and numerical examples

二、Relation of Tooth Numbers and Phase Angles Relation of Tooth Numbers.

The tooth numbers $N1$ and $N3$ of gears *1* and *3*（Fig. 1）are related as given in 8 and 10 as:

$$\frac{N_1+N_3}{n}=M \qquad (1)$$

where M is an integer number and n is the number of planetary gears.

Illustration of Installment of Planetary Gears.

1. Assume that planetary gear *2*(1) is installed on the carrier and is in mesh with

gears *1* and *3* as shown in Fig. 2（*a*）.

2. The neighboring planetary gear *2*(2) is in a position determined by angle f*c*52p/*n* （Fig. 2（*b*））

Angles u*m*1 (2) 2p/*N*1 uÞ 2p/*n* and u*m*3 (2) 2p/*N*3 uÞ 2p/*n* determine the positions of axes of symmetry of spaces of gears *1* and *3*, that are candidates for meshing with the tooth of planetary gear 2(2).

3. The meshing of planetary gear *2*(2) with gears *1* and *3* will start after gears *1*, 2(1) and *3* are rotated through angles d1 (2) and d3 (2) wherein carrier *c* is held at rest （Fig. 2（*b*））.

4. Drawings of Fig. 2（*b*） yield the following conditions of installment of gear *2*(*k*)(*k*52, . . . ,5)

5. Equations from

$$m_1^{(k)}\frac{2\pi}{N_1}-\delta_1^{(k)}=\frac{2\pi(k-1)}{n}(\delta_1^{(k)}<\frac{2\pi}{N_1}) \quad (2)$$

$$m_3^{(k)}\frac{2\pi}{N_1}-\delta_3^{(k)}=\frac{2\pi(k-1)}{n}(\delta_1^{(k)}<\frac{2\pi}{N_3}) \quad (3)$$

$$\frac{\delta_3^{(k)}}{\delta_1^{(k)}}=\frac{N_1}{N_3} \quad (4)$$

（2） to （4） yield the following relation

where *m*1 (*k*)1*m*3 (*k*) is an integer number. Equation （5） Confirms relation （1） mentioned above.

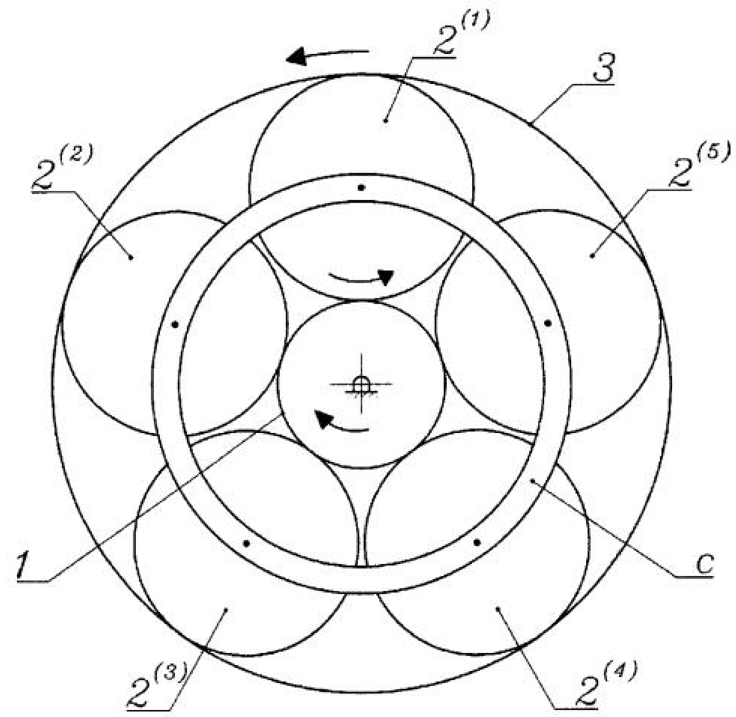


Fig. 1 Schematic of planetary gear train

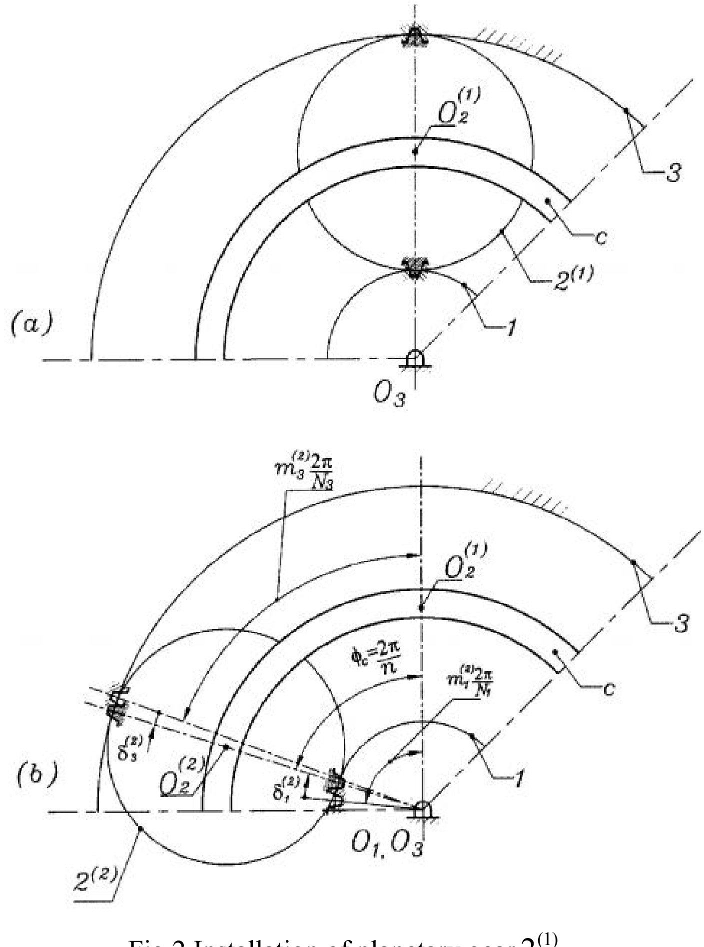


Fig.2 Installation of planetary gear $2^{(1)}$

6. Equations（2） and（3） and inequalities for d1 (*k*) and d3 (*k*) yield the following relations

$$m_1^{(k)} - \frac{(k-1)N_1}{n} < 1(k = 2,,,n) \qquad (6)$$

$$\frac{(k-1)N_3}{n} - m_3^{(k)} < 1 \qquad (7)$$

$$\delta_1^{(k)} = m_1^{(k)} \frac{2\pi}{N_1} - \frac{(k-1)2\pi}{n}(k = 2,,,n) \qquad (8)$$

$$\delta_3^{(k)} = \frac{(k-1)2\pi}{n} - m_3^{(k)} \frac{2\pi}{N_3}(k = 2,,,n) \qquad (9)$$

Note: Discussions above are true wherein planetary gears *2*(*i*) have an even number of teeth （Fig. 2（*a*）） or an odd tooth number.

Numerical Example 1. A planetary gear train with *N*1562, *N*35228, and *n*55 is considered. The results of computations are represented in Table 1.

Phase Angle of Planetary Gears. The phase angle is formed by the axis of symmetry of the tooth（space） with the respective line of center distance. The determination of the phase angle is based on the following considerations:

1、 Figure 2（*a*） shows the initial position of planetary gear *2*(1). The phase angle of planetary gear *2*(1) is equal to zero.

2 、Figure 2（*b*） shows the position of planetary gear *2*(1). The phase angle of *2*(2) is zero as well, but *2*(2) is not yet in mesh.

3、 We remind that to put gear *2*(2) into mesh with *1* and *3*, we make a turn of the sub-gear drive formed by gears *1*, *2*(1), and *3* through angles d1 (2) and d3 (2) （Fig. 2（*b*）） , wherein the carrier *c* and gear *2*(2) are held at rest. Then, the phase angle of *2*(1) will become different from zero, *2*(2) will be put into mesh with gears *1* and *3*, the phase angle of *2*(2) will be equal to zero.

4 、The initial position of *2*(1) with the phase angle zero is the reference position （Fig.2（*a*）） . Therefore, it is necessary to restore the zero phase angle of gear *2*(1). This can be done by turning of the sub-gear drive formed by gears *1*, *3*, *2*(1), and *2*(2) in the direction that is opposite to the direction of the previous turn of sub-gear drive formed by *1*, 2(1), and *3*. Figure 3 shows that after

such turn, the zero phase angle of *2*(1) is restored; gears *1*, *2*(2),and *3* are turned through angles m1 (2) , m2 (2) , and m3 (2) , respectively. Angles m1 (2) , m2 (2) , and m3 (2) indicate the deviation of the respective axes of tooth （space） symmetry of gears *1*, *2*(2), and *3* from the center distance *O*3*O*2 (¯2).

5 、The discussions above allow to determine the phase angles of gear *1* and planetary gears *2*(*k*) (*k*52, . . . ,5).

For instance, Fig. 4 represents to enlarged scale the orientation of spaces of gear *1* in

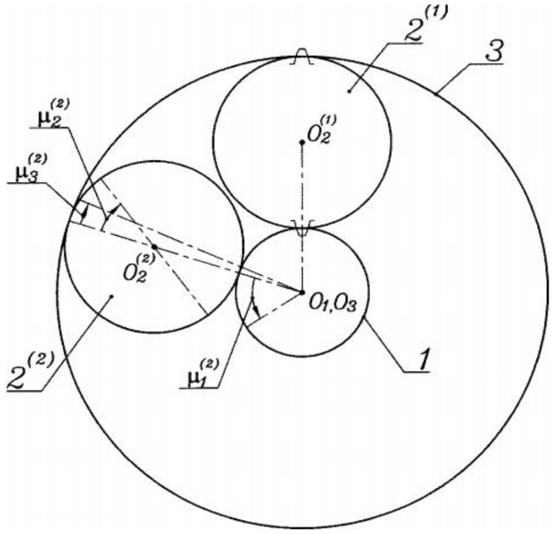


Fig. 3 Illustration of orientation of tooth 2, $3^{(2)}$,3spaces…of axes of symmetry of gears ..,3 with respect the center distance $o_3o_2^{(k)}$

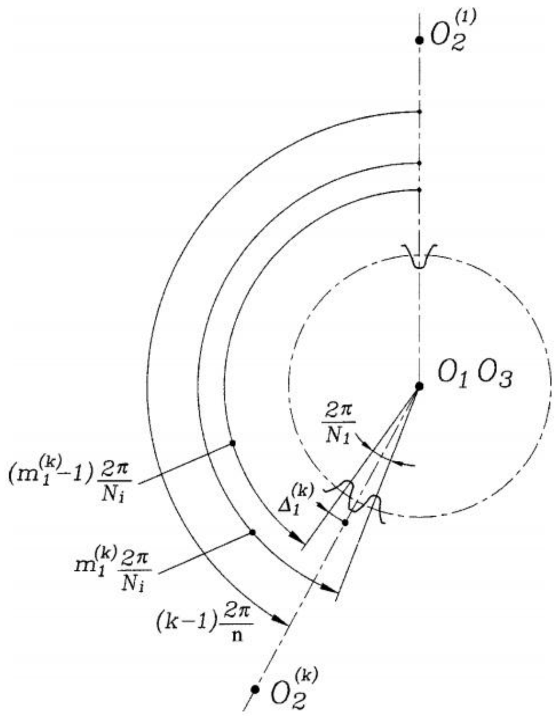


Fig. 4 For derivation of phase angle $\Delta_1^{(k)}$

the area of spaces of gear *1* identified by numbers 1 and *m*1 (*k*) (*k* 52, . . . 5), where *m*1 (*k*) is determined by Eq. （2）. The phase angle D1 (*k*) of gear *1* is formed by the line *O*3*O*2 (¯*k*) and the space number (*m*1 (*k*)21) of gear 1 that neighbors to *O*3*O*2 (¯*k*). The phase angle D1 (*k*) is measured from the center distance *O*3*O*2 (¯*k*) clockwise that is the direction of rotation of gear *1* wherein the carrier is held at rest （Fig. 1）.

Drawings of Fig. 4 yield the following equation for determination of the phase angle D1 (k)

Using the input data for Example 1, we obtainThe concept of the phase angle D1 (*k*) is used for computation of transmission errors （see section 5）.

$$\Delta_1^{(k)} = \frac{(k-1)2\pi}{n} - (m_1^{(k)} - 1)\frac{2\pi}{N_1}(k = 2,,,n) \qquad (10)$$

$$\Delta_1^{(2)} = \frac{2}{5.62}2\pi \text{；} \quad \Delta_1^{(3)} = \frac{2}{5.62}2\pi \text{；} \quad \Delta_1^{(4)} = \frac{2}{5.62}2\pi \text{；} \quad \Delta_1^{(5)} = \frac{2}{5.62}2\pi \text{；}$$

Gears of involute profiles are applied in the planetary train of existing design. The proposed new approach is based on the following modification of tooth geometry: （i） the planetary gears are double crowned for providing a predesign parabolic function of transmission errors and a localized bearing contact; （ii） Screw involute tooth surfaces of a small helix angle are applied for the sun and ring gears. The interaction of tooth surfaces of modified geometry enables to minimize and equalize the backlash for the improvement of load distribution between all of the planetary gears.

三、Modification of Geometry of Planetary Gears.

The modification is based on the following ideas: （i） profile crowning is obtained by application of a rack-cutter with parabolic profile （Fig. 5（*a*）） instead of a rack-cutter with straight line profile. Longitudinal crowning is obtained by plunging of the generating tool （see below）

$$r_b(u_c,\theta_c) = [a_a u_c^2 \quad u_c \quad \theta_c \quad 1]^T$$

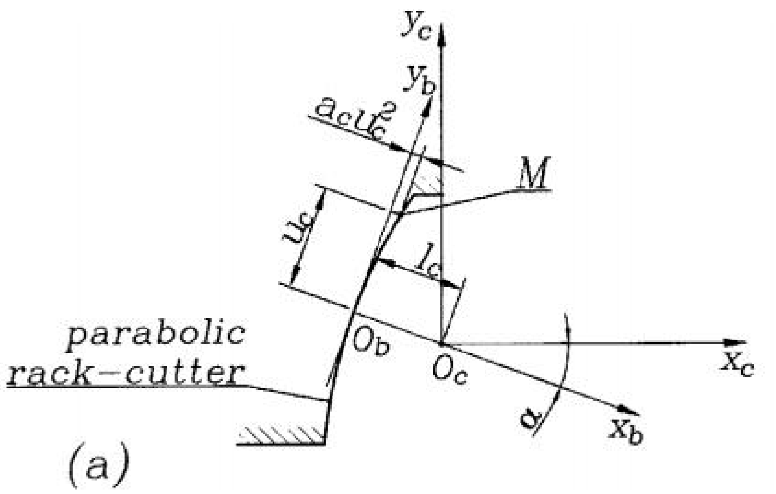
Coordinate systems *Sb* and *Sc* are rigidly connected to the parabolic rack-cutter represented in *Sb* by the vector equation

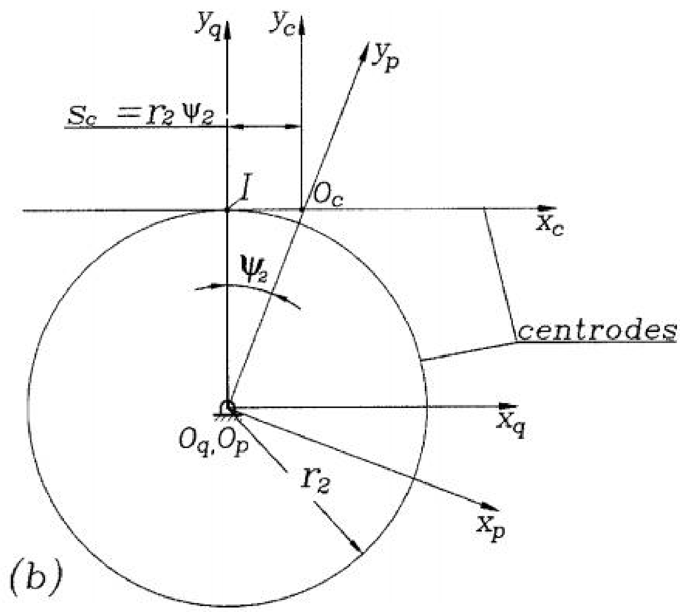
$$r_p(u_c,\theta_c,\varphi_2) = M_{pc}(\varphi)M_{cb}r_b(u_c,\theta_c) \qquad (12)$$

$$(\frac{\partial r_p}{\partial u_c} \times \frac{\partial r_p}{\partial \theta_c}) \times \frac{\partial r_p}{\partial \varphi_2} = f_{cp}(u_c, \varphi_2) = 0 \qquad (13)$$

Here: (*uc* ,u *c*) are the surface parameters (u *c* is measured along coordinate axis *zc*); *ac* is the parabola coefficient. The generation of the planetary gear is schematically shown in Fig. 5（*b*）. Coordinate system *Sp* is rigidly connected to the planetary gear; *Sq* is the fixed coordinate system; *sc*5*r*2c2 and c2 indicate the translational and rotational motion of the rack-cutter and the gear.

The profile crowned surface of the planetary gear is determined by the following equations [14]:





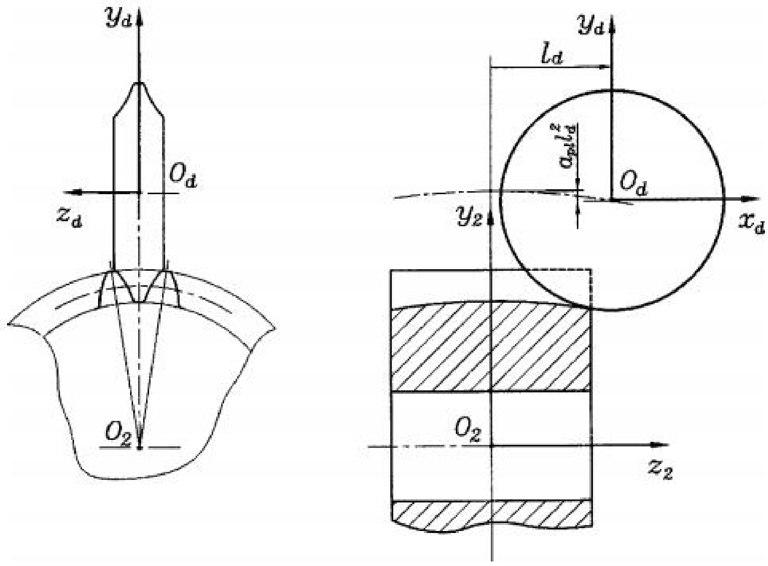


Fig. 5 Illustration of generation of profile crowned planetary

gear: （a）parabolic profile of applied rack-cutter; （b）schematic of generation

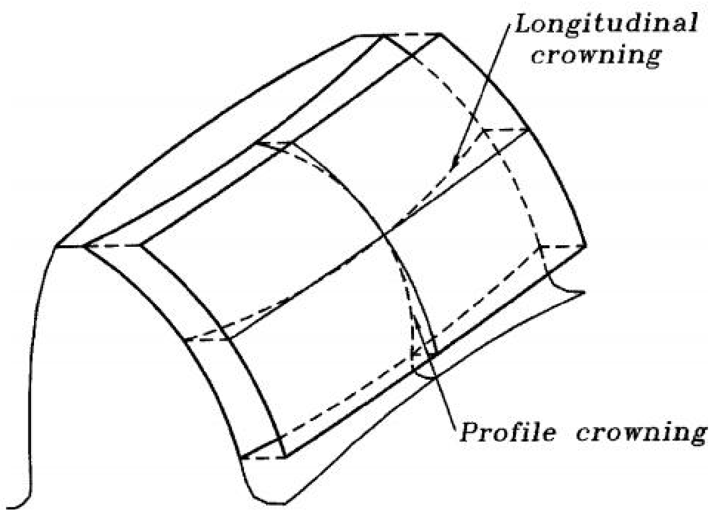


Fig. 6 Illustration of longitudinal crowning in planetary gear

$$r_2(u_c,\theta_d,l_d) = M_{2d}(l_d) r_d(u_c \times \theta_d) \quad （14）$$

$$f(u_c,\theta_d,l_d) = \left(\frac{\partial r_2}{\partial u_c} \times \frac{\partial r_2}{\partial \theta_d}\right) \times \frac{\partial r_2}{\partial l_d} = 0 \quad （15）$$

Here: matrices *pc* and *cb* describe coordinate transformation; vector function **r***p*(*uc* ,u *c* ,c2) represents in *S*2 the family of rack-cutter surfaces; Eq. （13） is the equation of meshing. Equations （12） and （13） determine in three-parameter form surface (*p* of profile crowned surface of the planetary gear.

Longitudinal crowning of the surface of the planetary gear is performed by plunging of a generating disk as shown in Fig. 6. The axial profile of the disk coincides with the transverse profile of surface (*p* , where (*p* is determined by Eqs. （12） and （13）.

The longitudinally crowned surface (2 of the planetary gear is determined by the equations

Here: vector function **r***d*(*uc* ,u *d*) represents the surface of the generating disk; *ld* and *aplld* 2 represent the translational motion of the disk and its plunge, respectively （Fig. 6）.

Figure 7 illustrates double-crowned tooth surface of a spur gear

Tooth Surfaces (1 and (3 of Gears *1* and *3*.

Tooth surface of (1 is a conventional screw involute surface of external meshing [14]. Respectively, tooth surface (3 is a conventional screw involute surface of internal meshing [14]. The direction and the helix angle of the screws are the *same* for both gears, *1* and *3*.

Interaction of Modified Tooth Surfaces.

Modification of tooth surfaces of gears *1*, *2*(*i*) (*i*51, . . . 5), *3* enables to reduce the backlash as shown schematically by Fig. 8. Due to errors of manufacture and installment, the backlash between the tooth surfaces of gears *1* and *3* and each of planetary gear of the set *2*(*i*)(*i*51, . . . 5) is not of the same magnitude. The main source of

backlash（or interference）is the error of angular installment of the planetary gear on the carrier.

Figure 8（*a*） shows the backlash D*x*(*i*) before regulation. Figure 8（*b*） shows the elimination of the backlash achieved by axial displacement D*z*(*i*) of the planetary gear. The described regulation by axial displacement D*z*(*k*) has to be accomplished for each planetary gear *2*(*i*) of the set (*i*51, . . . 5).

The regulation of backlash can be accomplished during the process of assemble by proper axial displacements of each of the planetary gears and requires only the test of achieved backlash.

四、Application of TCA for Determination of Transmission

Errors and Bearing Contact

TCA （Tooth Contact Analysis） is designated for simulation of meshing and contact of gear tooth surfaces of a misaligned gear drive. Two algorithms of TCA are represented in this paper: （i） for a conventional gear drive with two gears, and （ii） for a planetary gear drive.

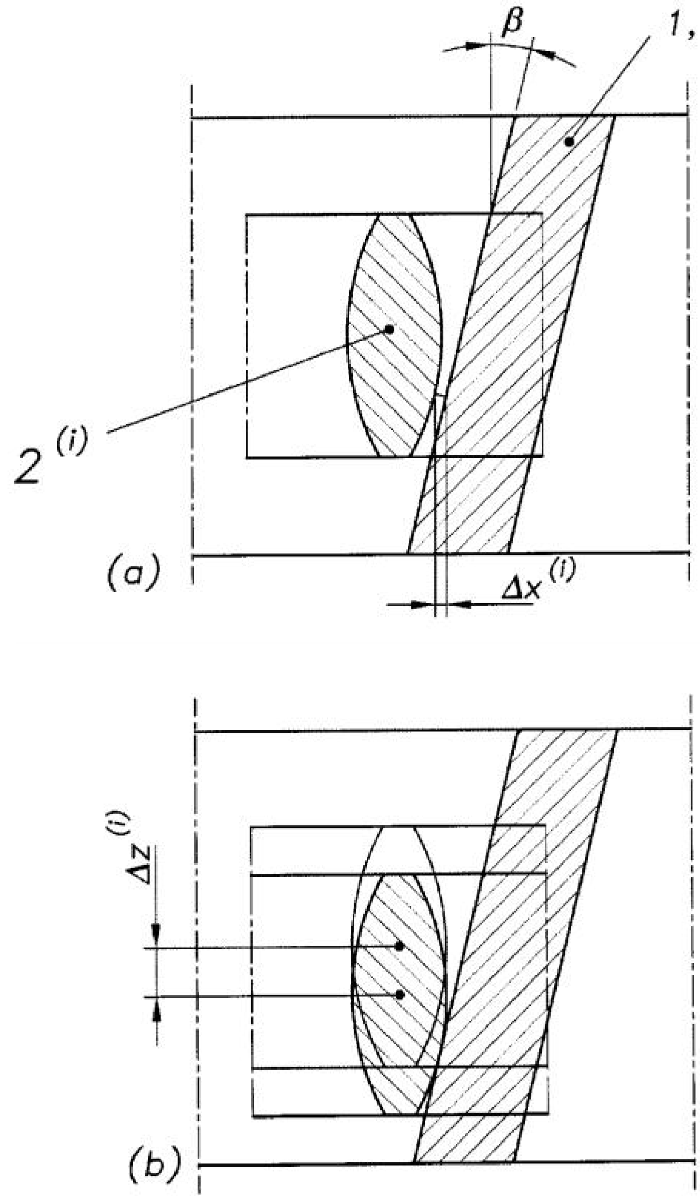


Fig. 8 Illustration of regulation of backlash: （a） backlash between gears 1, 3 and 2 before regulation; （b）elimination of backlash by axial displacement $\Delta z^{(i)}$

Conventional Gear Drive.

The tangency of surfaces is considered in a fixed coordinate system Sf. The TCA algorithm represents the conditions of continuous tangency of contacting surfaces (1 and (2 by the following vector equations

$$r_f^{(1)}(u_1,\theta_1,\phi_2)=r_f^{(2)}(u_2,\theta_2,\phi_2) \tag{16}$$

$$n_f^{(1)}(u_1,\theta_1,\phi_2)=u_f^{(2)}(u_2,\theta_2,\phi_2) \qquad (17)$$

Here: (*ui* ,u *i*) are the surfaces parameters; f*i* is the angle of rotation of gear *i* (*i*51,2); **r***f* (*i*)(*ui* ,u *i* ,f*i*) is the position vector of surface (*i* in fixed coordinate system *Sf* ; **n***f* (*i*)(*ui* ,u *i* ,f*i*) is the unit normal to surface (*i* in coordinate system *Sf* . Vector equations （16） and （17） yield five independent scalar equations represented as Note that vector equation（17）provides only two independent scalar equations since

$$f_i(u_1,\theta_1,\phi_1,u_2,\theta_2,\phi_2)=0 \qquad (i=1,,,5) \qquad (18)$$

The application of equation system （18） for TCA is an iterative process [15] that is performed as follows:

$$p^{(0)}=(u_1^{(0)},\theta_1^{(0)},\phi_1^{(0)},u_2^{(0)},\theta_2^{(0)},\phi_2^{(0)}) \qquad (19)$$

（i） It is assumed that there is a set of parameters
that satisfies equation system （18）.

（ii） One of the unknowns, say f1 , is chosen as an input parameter and the following inequality at *P*(0) is observed

（iii） Then, in accordance to the Theorem of Implicit Function System Existence [16], equation system （18） can be solved in the neighborhood of *P*(0) by functions

$$\Delta_1=\begin{vmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \phi_2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_5}{\partial u_1} & \frac{\partial f_5}{\partial \theta_1} & \frac{\partial f_5}{\partial u_2} & \frac{\partial f_5}{\partial \theta_2} & \frac{\partial f_5}{\partial \phi_2} \end{vmatrix}\neq 0 \qquad (20)$$

（iv） Applying the iterative process mentioned above for the area 2p/*N*1 <f1<p/*N*1 , it becomes possible to determine functions （21） for the whole cycle of meshing determined by f152p/*N*1 .

（v） Using transmission function f2(f1), it becomes possible to determine the function of transmission errors as （vi） Path of contact on surface (*i* (*i*51,2) is determined as

where **r***i*(*ui*(f1),u *i*(f1)) is the vector function that represents (*I* in coordinate system *Si* .

$$\mathbf{r}_i(u_i(\phi_1),\theta_i(\phi_1)) \tag{23}$$

Planetary Gear Train.

Application of TCA for determination of transmission errors of the planetary gear train is a more complex problem and requires the following steps:

Step 1: Tooth surfaces of gears *1*, *2*(*i*) and *3* are represented in fixed coordinate system *S*3 .

Step 2: The rotation of gears *1* and *2*(*i*) is determined by three parameters f1 , f2*c* (*i*) , and f*c* . Here: f1 is the angle of rotation of gear *1*, f2*c* (*i*) is the angle of rotation of planetary gear *2*(*i*) with respect to the carrier *c*, and f*c* is the angle of rotation of the carrier.

Step 3: Conditions of tangency of gears *1* and *2*(*i*), and gears *2*(*i*) and *3* provide ten independent equations that are similar to Eqs. （18）. These equations contain eight surface parameters of

gears *1*, 2(*i*), and *3* and three motion parameters f1 , f2*c*

(*i*) , and f*c* . Considering f1 as the input parameter, we may obtain from the TCA computer program the transmission function f*c*(f1) and then determine the function of transmission errors of the misaligned planetary gear train.

The solution of ten nonlinear equations can be simplified by

representing them as two sub-systems of five equations each and then applying an iterative process of solution. Considering the planetary gear train as a multi-system of rigid bodies and applying TCA, we may determine at any instant which of the planetary gear of the set of five is in mesh with gears *1* and *3*.

五、**Functions of Transmission Errors of Sub-Gear Drives**

Sub-gear drives (*1,2*(*i*)), (*2*(*i*),*3*), and (*1,2*(*i*),*3*) of the planetary gear train are considered. Transformation of rotation of the sub-gear drives is performed whereas the carrier is held at rest. Applying TCA for the sub-gear drives, it becomes possible to determine the functions of transmission errors and as well as the backlash. Then, it becomes possible to minimize and equalize the backlash of five planetary gears by regulation.

The resulting function of transmission errors of the sub-gear drive (*1,2*(*i*),*3*) is determined as

$$\Delta\phi_3(\phi_1) = \phi_3(\phi_2(\phi_1)) - \frac{N_1}{N_3}\phi_1$$

$$\Delta\phi_3(\phi_1) \cong \frac{N_2}{N_3}\Delta\phi_2(\phi_1) + \Delta\phi_3\left(\frac{N_1}{N_2}\phi_1\right)$$