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ORIGINAL ARTICLE

Pseudo-constructal theory for shape optimization of mechanical structures

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Abstract This work gives some applications of a pseudo-constructal technique for shape optimization of mechanical structures. In the pseudo-constructal theory developed in this paper, the main objective of optimization is only the minimization of total potential energy. The other objectives usually used in mechanical structures optimization are treated like limitations or optimization constraints. Two applications are presented; the first one deals with the optimization of the shape of a drop of water by using a genetic algorithm with the pseudo-constructal technique, and the second one deals with the optimization of the shape of a hydraulic hammer's rear bearing.

Keywords Shape optimization · Constructal · Genetic algorithms

1 Introduction

This paper introduces a pseudo-constructal approach to shape optimization based on the minimization of the total potential energy. We are going to show that minimizing the total potential energy of a structure to find the optimal shape might be a good idea in some cases. The reference to the constructal theory can be justified in some way for the following reasons.

According to Bejan [1], shape and structure spring from the struggle for better performance in both engineering and nature; the objective and constraints principle used in engineering is the same mechanism from which the geometry in natural flow systems emerges. Bejan [1] starts with the design and optimization of engineering systems and discovers a deterministic principle for the generation of geometric form in natural systems. This observation is the basis of the new constructal theory. Optimal distribution of imperfection is destined to remain imperfect. The system works best when its imperfections are spread around so that more and more internal points are stressed as much as the hardest working parts. Seemingly universal geometric forms unite the flow systems of engineering and nature. Bejan [1] advances a new theory in which he unabashedly hints that his law is in the same league as the second law of thermodynamics, because a simple law is purported to predict the geometric form of anything alive on earth.

Many applications of the constructal theory were developed in fluids mechanics, in particular for the optimization of flows [2–10]. On the other hand, there exists, to our knowledge, little examples of applications in solids or structures mechanics. So we have at least half of the references to papers in fluid dynamics (most of the same author), because the constructal method was developed first by the same author, Adrian Bejan, with only references to papers in fluid dynamics. The constructal theory rests on the assumption that all creations of nature are overall optimal compared to the laws which control the evolution and the adaptation of the natural systems. The constructal principle consists of distributing the imperfections as well as possible, starting from the smallest scales to the largest. The constructal theory works with the total macroscopic structure starting from the assembly of elementary structures, by complying with the natural rules of optimal distribution of the imperfections. The objective is the research of lower cost.

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However, a global and macroscopic solution for the optimization of mechanical structures having least cost as the objective can be very close to the constructal theory, from where the term pseudo-constructal comes. The constructal theory is a predictive theory, with only one single principle of optimization from which all rises. The same applies to the pseudo-constructal step which is the subject of this article. The single principle of optimization of the pseudo-constructal theory is the minimization of total potential energy. Moreover, in our examples presented hereafter, the pseudo-constructal principle will be associated with a genetic algorithm, with the result that our optimization will be very close to the natural laws.

The objective of this paper is thus to show how the pseudo-constructal step can apply to the mechanics of the structures, and in particular to the shape optimization of mechanical structures. The basic idea is very simple: a mechanical structure in a balanced state corresponds to a minimal total potential energy. In the same way, an optimal mechanical structure must also correspond to a minimal total potential energy, and it is this objective which must intervene first over all the others. It is this idea which will be developed in this article.

Two examples will be presented thereafter. The idea to minimize total potential energy in order to optimize a mechanical structure is not brand new. Many papers already deal with this problem. What is new, is to make this approach systematic. The only objective of optimization becomes the minimization of energy.

In Gosling [11], a simple method is proposed for the difficult case of form-finding of cablenet and membrane structures. This method is based upon basic energy concepts. A truncated strain expression is used to define the total potential energy. The final energy form is minimized using the Powell algorithm. In Kanno and Ohsaki [12], the minimum principle of complementary energy is established for cable networks involving only stress components as variables in geometrically nonlinear elasticity. In order to show the strong duality between the minimization problems of total potential energy and complementary energy, the convex formulations of these problems are investigated, which can be embedded into a primal-dual pair of second-order programming problems. In Taroco [13], shape sensitivity analysis of an elastic solid in equilibrium is presented. The domain and boundary integral expressions of the first and second-order shape derivatives of the total potential energy are established. In Warner [14], an optimal design problem is solved for an elastic rod hanging under its own weight. The distribution of the cross-sectional area that minimizes the total potential energy stored in an equilibrium state is found. The companion problem of the design that stores the maximum potential energy under the same constraint conditions is also solved. In Ventura [15], the problem of boundary conditions enforcement in meshless methods is solved. In Ventura [15], the moving least-squares approximation is introduced in the total potential energy functional for the elastic solid problem and an augmented Lagrangian term is added to satisfy essential boundary conditions.

The principle of minimization of total potential energy is in addition at the base of the general finite elements formulation, with an aim of finding the unknown optimal nodal factors [16].

2 The methods used

In the pseudo-constructal theory developed in this paper, the main objective of optimization is only the minimization of total potential energy. The other objectives usually used in mechanical structures optimization are treated here like limitations or optimization constraints. For example, one may have limitations on the weight, or to not exceed the value of a stress.

The idea which will be developed in this paper is thus very simple. A mechanical structure is described by two types of parameters: variables known as discretization variables (for example, degrees of freedom in displacement for finite elements method), and geometrical variables of design (for example parameters which make it possible to describe the mechanical structure shape). Total potential energy depends on an implicit or explicit way of determining discretization and design variables at the same time. One thus will carry out a double optimization of the mechanical structure, compared to the discretization and design variables; the objective being to minimize total potential energy overall. Clearly, the problem of optimization of a mechanical structure will be addressed by the following approach:

- Objective: to minimize total potential energy
- Variables of optimization: concurrently determining discretization variables (in the case of a traditional use of the finite element method in mechanics of structures), and design variables describing the shape of the structure
- Optimization limitations:
  - Weight or volume
  - Displacements or strains
  - Stresses
  - Frequencies

The problem of optimization of a mechanical structure will be solved in the following way, while reiterating on these stages, if needed (according to the nature of the problem):

Stage 1 Minimization of the total potential energy of the mechanical structure compared to the only discretization variables of the structure (degrees of freedom in finite elements). It acts here as an optimization without optimization limitations. The only limitations at this stage are of purely mechanical origin, and relate to the boundary conditions and to the external efforts applied to the structure.

In this stage 1, the design variables remain fixed, and one obtains the implicit or explicit expressions of the degrees of freedom according to the design variables (which can be the variables which make it possible to describe the shape, in the case of a shape optimization, for example). One will see in the examples of the following part that these expressions can be explicit or implicit and which is the suitable treatment following the cases. In the case of a finite elements method of calculation, this stage 1 is the basis of finite elements calculation to obtain the degrees of freedom of the mechanical structure. Indeed, in finite elements, displacements with the nodes of the mechanical structure mesh are obtained by minimization of total potential energy [16].

Stage 2 The expressions of the degrees of freedom of the mechanical structure according to the design variables obtained previously are then injected into the total potential energy of the mechanical structure (one will see in the second example of the following part how one treats the case where the degrees of freedom are implicit functions of the design variables). One then obtains an expression of the total potential energy which depends only on the design variables (in explicit or implicit form).

Stage 3 One then carries out a second and new minimization of the total potential energy obtained in the preceding form, but this time compared to the design variables while respecting the technological limitations or the optimization constraints of the problem. This method can be applied with more or less facility according to the nature of the problem. It is clear, for example, that if the discretization variables can be expressed in an explicit way according to the design variables, the setting in of stages 2 to 3 is immediate, and without iterations.

If the discretization variables cannot be expressed in an explicit way according to the design variables, or if the topology of the structure is not fixed, or if the behavior is not linear, it will be necessary to proceed by successive iterations on stages 1 to 3. It is the case of the examples presented in the following part, and one will see on this occasion which type of strategy one can adopt for these iterations. To summarize, in the pseudo-constructal step, the main objective is only the minimization of total potential energy, the other possible objectives are treated like limitations or optimization constraints.

The optimization method used for our examples is GA (genetic algorithm), as described in [17]. Examples with similar instructional value can also be found in many books, e.g. in [18]. This evolutionary method is very convenient for our pseudo-constructal method. The author has worked extensively in GAs and published in some reputed journals on this topic [19–31]. As the topic of GAs is still relatively new in the structural mechanics community, we provide here some details of exactly what is used in this GA. A multiple point crossover is used rather than a single point crossover. The selection scheme used at each generation is entirely stochastic. For our examples, the number of generations is equal to that used for convergence. The results provided for our examples were consistently reproduced by using different seeds in the GA. It has been proved that a rather standard genetic algorithm is sufficient for our examples.

3 Examples

Even though potential energy may be a good measure for some optimizations, potential energy is not what gives the shape to a water droplet, nor defines the optimal shape for a hammer, which is why potential energy is not the only objective; but the optimization problem is a multiobjective one and the objective functions for the two examples are then clearly formulated.

3.1 Example 1: optimization of the shape of a drop of water

The first test example is the optimization of the shape of a drop of water (Fig. 1). This problem is equivalent to an equal resistance tank calculated by the membrane theory. The objective is to see if the pseudo-constructal theory gives the nature's optimum design.

3.1.1 The methods used

The geometry of the drop of water is defined by the generating line of a thin axisymmetric shell. This line is described by successive straight or circular segments described in a given sense and defined by input data of master point coordinates and radius values. The initial data are a set of nodal points connected by straight segments. Each nodal point is identified by its two cylindrical

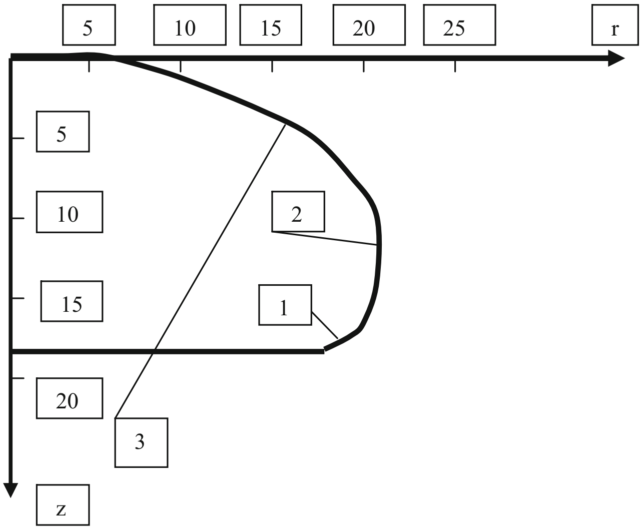

Fig. 1 Optimization of the shape of a drop of water

coordinates (r, z), and a real R which represents the radius of the circle tangent to the two straight segments intersecting at the point. The other computer calculations give the coordinates of any boundary point and especially the tangent points necessary to define the circular arc lengths. The design of the drop of water is described by three arcs of circles as indicated in Fig. 1.

Analysis is performed by the finite element method with three-node parabolic elements using the classical Love-Kirchoff shell theory. An automatic mesh generator creates the finite element mesh of each straight or circular segment considered as a macro finite element.

The objective is to obtain a shape for the drop of water giving rise to a minimum total potential energy (which is the main objective) and an equal resistance tank (which is the only constraint or limitation of the problem).

In fact, for the drop of water problem, the goal is a multi-objective one, the two objectives (f_1=minimum total potential energy and f_2=equal resistance) are combined in a multi-objective: $f=f_1+f_2$.

The constraint or limitation of the problem is taken into account by a penalization of the total potential energy as indicated in Marcelin et al. [19].

3.1.2 The results

The design of the drop of water is described by three arcs of a circle (Fig. 1). Their centers and radius are the design variables. So, there are nine design variables: $r1$, $z1$, $R1$ for circle 1; $r2$, $z2$, $R2$ for circle 2; and $r3$, $z3$, $R3$ for circle 3. In the genetic algorithm, each of these design variables is coded by three binary digits.

The tables of coding-decoding will be the following:

For $r1$:

000	001	010	011	100	101	110	111
16	16.5	17	17.5	18	18.5	19	19.5

For $z1$:

000	001	010	011	100	101	110	111
15	15.5	16	16.5	17	17.5	18	18.5

For $R1$:

000	001	010	011	100	101	110	111
−0.050	−0.055	−0.060	−0.065	−0.070	−0.075	−0.080	−0.085

For $r2$:

000	001	010	011	100	101	110	111
13	13.25	13.5	13.75	14	14.25	14.5	14.75

For $z2$:

000	001	010	011	100	101	110	111
12	12.1	12.2	12.3	12.4	12.5	12.6	12.7

For $R2$:

000	001	010	011	100	101	110	111
−7.4	−7.5	−7.6	−7.7	−7.8	−7.9	−8	−8.1

For $r3$:

000	001	010	011	100	101	110	111
3.7	3.8	3.9	4	4.1	4.2	4.3	4.4

For $z3$:

000	001	010	011	100	101	110	111
21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8

For $R3$:

000	001	010	011	100	101	110	111
−18.5	−19	−19.5	−20	−20.5	−21	−21.5	−22

All these binary digits are put end to end to form a chromosome length of 27 binary digits.

GA is run for a population of 30 individuals, a number of generations of 50, a probability of crossing of 0.8, and a probability of mutation of 0.1.

The optimal solution corresponds to the chromosome
100 100 011 011 010 011 100 011 101
which gives the solution of Fig. 1, for which:

- $r1=18$, $z1=17$,and $R1=-0.065$
- $r2=13.75$, $z2=12.2$ and $R2=-7.7$
- $r3=4.1$, $z3=21.4$ and $R3=-21$

It is very close to the nature's optimal solution for the shape of a drop of water. The model of the water drop modelled by three arcs of a circle is imperfect. However, the constructal theory optimizes the imperfections, and finds the nearest solution to that of nature. So, the constructal principle consists of distributing the imperfections as well as possible.

3.2 Example 2: optimization of the shape of an axisymmetric structure

In this part, the very localized optimization of the rear bearing of a hydraulic hammer is presented. The bearing in question (Fig. 2) breaks after relatively few cycles of operation.

For axisymmetric structures, analysis is performed by the finite element method in which the special character of a GA optimization process has been considered to ease the calculations and to save computer time. First, because just a few parts of the structure must often be modified, the substructure concept is used to separate the "fixed" and the "mobile" parts. The fixed parts are calculated twice: once at the beginning and also at the end of the optimization process. Only the reduced stiffness matrices of these substructures are added to the matrices of the mobile parts. Related to this division, an automatic generator creates the finite element mesh of each substructure considered as a macro finite element. These macro elements are either triangular (six nodes) or quadrilateral (eight nodes). Following a well-known technique, the same subdivision is used in the parent space to obtain the mesh itself, which is obviously made out of the same types of elements. During the optimization process this mesh is controlled and a new discretization can be chosen if necessary.

To summarize, the optimization problem is the following:

Objective function Minimization of the total potential energy. It is important to note that another important objective (the minimization of the maximum value of the Von Mises equivalent stress along the mobile contour) is taken here as a constraint of the problem. This second objective is necessary to achieve the minimization of the rear bearing of the hydraulic hammer .

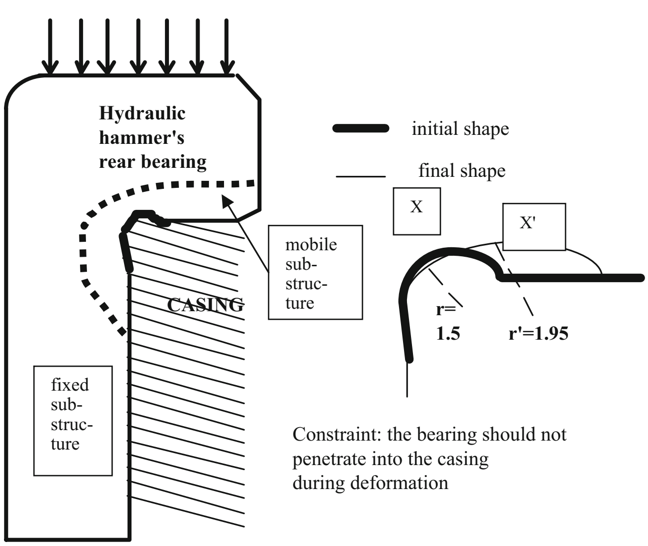

Fig. 2 Optimization of the shape of a hydraulic hammer's rear bearing

Design variables The design variables are radius r and width X near the radius (Fig. 2).

Constraints The side constraints are established in such a way that only small changes in geometry are allowed. They take into account the technological constraints. They are included in the coding of the design variables. Another important constraint is that the maximum value of the Von Mises equivalent stress along the mobile contour must not exceed a certain value. The constraints are taken into account by a penalization of the total potential energy as indicated in [19].

The tables of coding-decoding are the following:

For r:

0000	0001	0010	0011	0100	0101	0110	0111
1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65
1000	1001	1010	1011	1100	1101	1110	1111
1.70	1.75	1.80	1.85	1.90	1.95	2.00	2.05

For X:

0000	0001	0010	0011	0100	0101	0110	0111
2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6
1000	1001	1010	1011	1100	1101	1110	1111
6.0	6.4	6.8	7.2	7.6	8.0	8.4	8.8

All these binary digits are put end to end to form a chromosome length of eight binary digits.

The GA is run for a population of 12 individuals, a number of generations of 30, a probability of crossing of 0.5, and a probability of mutation of 0.06.

The optimal solution corresponds to the chromosome

1101 1000

which gives the solution of Fig. 2, for which:

r=1.95, X=6.0

The automatic optimization of the shape of this product has, simply by a small modification of shape, which is difficult to predict other than by calculation (increased radius, decreased width), considerably improved the mechanical durability of the bearing: the over-stress being reduced by 50%.

4 Discussion

The two examples in this paper may prove the truth of the pseudo-constructal theory. The first one was the shape optimization of an axisymmetric membrane drop-shaped shell (water droplet). This structure is in pure tension. Sure enough, minimizing the total potential energy of this structure over all possible variables leads to a shape which is fully-stressed and very similar to what nature does. But the second example proves that the minimum energy formulation may not only work in the simplest of cases, of pure tension structures, but also for more complicated structures with bending, shear or torsion stresses. The condition for this is to add to the problem secondary objectives (usually used in shape optimization) as limitations or optimization constraints.

Nevertheless, the assertion at the heart of pseudo-constructal theory, that minimizing the total potential energy of mechanical structure over all possible variables is what nature does, is not entirely true. Neither nature nor engineering is so simplistic, and many years of research into how nature designs structures has shown that even in the most simple instances multiple criteria are at work in complicated ways leading to delicate compromises, so the necessity to add others criteria or constraints to the optimization problems is evident. Minimizing the total potential energy is just a general principle to start the optimization process.

5 Conclusion

An interesting approach has been introduced to perform shape optimization of mechanical structures. In the pseudo-constructal theory developed in this paper, the main objective of optimization is only the minimization of total potential energy. The other objectives usually used in shape optimization are treated here like limitations or optimization constraints. It gives good results for our examples.

References

1. Bejan A (2000) Shape and structure, from engineering to nature. Cambridge University Press, Cambridge, UK
2. Bejan A (1997) Constructal-theory network of conducting paths for cooling a heat generating volume. Int J Heat Mass Trans 40:799–816
3. Bejan A (1997) How nature takes shape. Mech Eng 119(10):90–92
4. Arion V, Cojocari A, Bejan A (2003) Constructal tree shaped networks for the distribution of electric power. Energy Convers Manag 44:867–891
5. Nelson RA, Bejan A (1998) Constructal optimization of internal flow geometry in convection. J Heat Trans 120:357–364
6. Rocha LAO, Lorente S, Bejan A (2002) Constructal design for cooling a disc-shaped area by conduction. Int J Heat Mass Trans 45:1643–1652
7. Bejan A, Ikegami Y, Ledezma GA (1998) Constructal theory of natural crack pattern formation for fastest cooling. Int J Heat Mass Trans 41:1945–1954
8. Bejan A (2000) From heat transfer principles to shape and structure in nature: constructal theory. J Heat Trans 122:430–449
9. Rocha LAO, Bejan A (2001) Geometric optimization of periodic flow and heat transfer in a volume cooled by parallel tubes. J Heat Trans 123:233–239
10. Wechsatol W, Lorente S, Bejan A (2002) Optimal tree-shaped networks for fluid flow in a disc-shaped body. Int J Heat Mass Trans 45(2):4911–4924
11. Gosling PD (1998) A discrete energy formulation for the definition of optimal tension structures. Struct Optim 16(1):76–80
12. Kanno Y, Ohsaki M (2003) Minimum principle of complementary energy of cable networks by using second-order cone programming. Int J Solids Struct 40(17):4437–4460
13. Taroco E (2000) Shape sensitivity analysis in linear elastic fracture mechanics. Comput Meth Appl Mech Eng 188(4):697–712
14. Warner WH (2003) The maximum potential energy of elastic rods under axial gravitational load. Int J Solids Struct 40(3):611–631
15. Ventura G (2002) An augmented Lagrangian approach to essential boundary conditions in meshless methods. Int J Numer Meth Eng 53(4):825–842
16. Zienkiewicz OC (1971) The finite element method in engineering science. McGraw-Hill, London
17. Goldberg DE (1989) Genetic algorithm in search, optimization, and machine learning. Addison-Wesley, Boston
18. Bentley PJ (1999) Evolutionary design by computers. Morgan Kaufmann, New York
19. Marcelin JL, Trompette P, Dornberger R (1995) Optimisation of composite beam structures using a genetic algorithm. Struct Optim 9:236–244
20. Marcelin JL (2001) Genetic optimization of stiffened plates and shells. Int J Numer Meth Eng 51(9):1079–1088
21. Marcelin JL (1999) Evolutionary optimization of mechanical structures. Eng Optim 31(5):571–588
22. Marcelin JL (1999) Evolutionary optimisation of mechanical structures: towards an integrated optimization. Eng Comput 15:326–333
23. Marcelin JL (2001) Genetic search applied to selecting support positions in machining of mechanical parts. Int J Adv Manuf Technol 17(5):344–347
24. Marcelin JL (2001) Genetic optimization of gears. Int J Adv Manuf Technol 17(12):910–915
25. Marcelin JL (2002) Genetic optimization of supports in vibrating structures. Eng Optim 34(1):101–107
26. Marcelin JL (2002) Genetic optimization of stiffened plates without the FE mesh support. Int J Numer Meth Eng 54(5):685–694
27. Marcelin JL (2004) A metamodel using neural networks and genetic algorithms for integrated optimal design of mechanisms. Int J Adv Manuf Technol 24(9–10):708–714
28. Marcelin JL (2005) Using genetic algorithms for the optimization of mechanisms. Int J Adv Manuf Technol 27(1–2):2–6
29. Marcelin JL (2006) Genetic optimization of vibrating stiffened plates. Struct Eng Mech 24(5):529–541
30. Marcelin JL (2007) Optimization of the cooling regime of hot-rolled complex beams. Int J Adv Manuf Technol 32(7–8):711–718
31. Marcelin JL (2007) Genetic optimal shape design of thin axisymmetric shells and axisymmetric structures. Comput Assist Mech Eng Sci 14(1):43–51