Available online at www.sciencedirect.com
SCIENCE DIRECT®
Int. J. Production Economics 98 (2005) 172–178
international journal of production economics
www.elsevier.com/locate/ijpe

Optimal design of rack structure with modular cell in AS/RS

Young Hae Lee*, Moon Hwan Lee, Sun Hur

Department of Industrial Engineering, Hanyang University, Ansan, Kyunggi-do, 425-791, Republic of Korea

Available online 17 February 2005

Abstract

In this paper, the model of AS/RS with the rack of modular cells is proposed first. In general, under the concept of unit load, Automated Storage/Retrieval Systems (AS/RS) has the rack of equally sized cells. Many authors have studied the design of AS/RS with the rack of equally sized cells. However, they are inadequate and inefficient in meeting the various sizes of customers' demands in today's business environment. Higher utilization and flexibility of warehouse storage can be achieved by using AS/RS with the rack of modular cells. The best size of modular cell is determined as a decision variable and the effectiveness of the proposed model is also presented. The model developed in this research, is one type of AS/RS that is more flexible to the size and has higher space utilization than those of existing rack structure, could be a very useful alternative for the storage of different unit load sizes.

Keywords: AS/RS; Modular cell; Equally sized cells; Flexibility and utilization

1. Introduction

Automated Storage/Retrieval Systems (AS/RS) is widely used in numerous manufacturing factories and distribution centers in the world. A typical AS/RS is composed of multiple parallel aisles of racks with storage cells (slots), a storage/retrieval (S/R) machine for each aisle, and input/output (I/O) station. The S/R machine moves simultaneously in horizontal and vertical direction in order to reduce the travel time, which is called as Tchebychev travel. The S/R machine can be operated under single and/or dual command. In a single command, only one operation of storage or retrieval of item is conducted. However, in a dual command both storage and retrieval of items are conducted during one cycle of S/R machine with an interleaving.

There are various types of AS/RS with equally sized cells according to the size and volume of items to be handled, storage and retrieval methods, and interaction of a S/R machine with the worker such as unit load AS/RS, mini-load AS/RS, man-on-board AS/RS, automated item-retrieval system, and deep-lane AS/RS (Groover, 1987). The design of AS/RS involves the determination of

*Corresponding author. Tel.: +82 31 400 5262; fax: +82 31 602 7730.
E-mail address: yhlee@hanyang.ac.kr (Y.H. Lee).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2004.05.018

the number of S/R machines, their horizontal/vertical velocities and travel times, the physical configuration of the storage racks, etc. The design of warehouses has been studied basically with two approaches: analytical optimization methods and simulation. As for the analytical methods, Roberts and Reed (1972) presented an optimization model to determine the warehouse bay configuration that minimizes the cost of handling and construction, ignoring the constraints on handling capacity of equipment and building sites. Karasawa et al. (1980) developed a non-linear mixed integer programming (MIP) for a deterministic model of an AS/RS to minimize the total cost. Their cost model included storage racks, storage building, handling machines and land. The decision variables involved were the number of cranes and the height and the length of rack. Optimization was performed as a function of sufficient storage volume for all items and sufficient the number of cranes to serve all storage and retrieval requests. Ashayeri et al. (1985) presented a mathematical model for the calculation of the optimal number of cranes and the optimal width and length of the warehouse subject to constraints on the constant crane velocities, the throughput and the length and width of building site.

In the simulation methods, Bafna and Reed (1972) proposed a simulation program to evaluate the alternative design of high-rise automated warehouse systems. Rosenblatt and Roll (1984) applied a search procedure to a simulation model of an automated warehousing system to find an optimum solution for minimizing the total cost of construction and operation. Perry et al. (1984) developed an optimum-seeking procedure and applied it interactively to simulated models of automated warehousing systems. Rosenblatt et al. (1993) proposed a recursive heuristic optimization–simulation model for obtaining optimal designs parameters for an AS/RS. This model finds the physical characteristics of the AS/RS, but the relationship between dimension of rack and capacity of S/R machine that could affect its performance was not considered in their model.

Warehouse designs may be evaluated in several ways, and different measures of effectiveness have been considered by many authors. The most common ones are: throughput as measured by the number of pallets or orders handled per day (Bafna and Reed, 1972; Perry et al., 1984), average waiting time per customer/order or percentage of customers/orders waiting to be served beyond a prespecified figure (Perry et al., 1983; Azadivar, 1989), are average travel time of a crane per single/dual command (Graves et al., 1977; Han et al., 1987; Rosenblatt and Eynan, 1989; Lee, 1997). In most cases, measures of effectiveness are affected by the physical layout design and the method of operation of the AS/RS.

As explained above, many authors have studied the optimal design of AS/RS with the rack of equally sized cells for using the concept of unit load. However, in terms of the flexibility of storage capability, the existing rack configuration using the concept of unit load is inefficient and inadequate for the storage of various types and various sizes of customers' demands. Moreover, if the various sizes of products are to be stored in existing systems, the space utilization will be considerably decreased due to the increase of lost space in each cell.

For the purpose of coping with the business environment that is changing rapidly, Lee et al. (1999) proposed the model of AS/RS with the rack of unequal sized cells. That is, in their model cells within the zone have the same size, but the sizes of cells in the different zones are different in height such that the rack can hold various types of load. Their model will be a good alternative for coping with those problems described above. However, if the quantity of storage demands for different sizes' products fluctuates in large, even the model proposed by Lee et al. (1999) will not basically be able to overcome inflexibility and low space utilization problems in existing rack structure of AS/RS.

In order to resolve these drawbacks, in this paper the model of AS/RS with the rack of modular cells is proposed first. Flexibility and space utilization of proposed model are compared to those of existing model through numerical examples. The remainder of this paper is organized as follows: we define the rack structure with modular cells and describe assumptions used in this research in Section 2. Section 3 presents the optimization model for the determination of modular cell size in height. In Section 4, the effectiveness of proposed model is analyzed with respect to the flexibility, space utilization and cost. Finally, a summary of the research is presented in Section 5.

2. Assumptions and notations

The structure of rack with modular cells to be modeled in this paper is depicted in Fig. 1. The rack structure with modular cells, considered in this research, is defined as follows: The rack can handle different sized loads in height, while it maximizes space utilization. The rack consists of modular cells, which allows the loads to occupy more than one cell according to their height. Modular cells have only 4 load-arms (brackets) and they have an open structure with same height (refer the Fig. 1). It is called ''opening cell'' because the top and bottom of the cell are not closed from bottom to top of the rack. Thus, the size of modular cell is determined by the size of products so as to minimize total lost spaces in the system.

2.1. Assumptions

The following assumptions are made in this research.

(1) The warehouse is divided into several aisles with racks on both sides. Thus, there are double racks between aisles and single rack along the walls. The rack has N_l levels and N_b bays of modular cells.
(2) The number of S/R machines is equal to the number of aisles. No traversing is allowed between aisles, so they only can serve one aisle.
(3) There are no technical problems for the construction of rack with modular cells proposed. However, applying this model to the storage of heavy product may be limited.
(4) The products of different size and various types can be stored in the system, if the size of width and length of product is fitted to that of modular cell.
(5) The information for the size and expected storage volume of each product is known in advance. However, if the storage volume information of products is unknown, it is assumed that they have the same values, which are calculated by $n_i = n/k$.

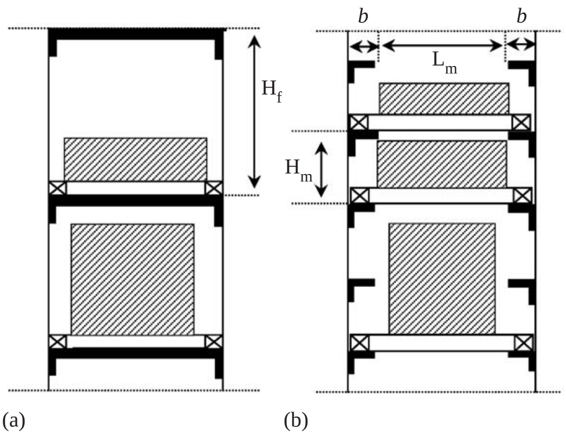

Fig. 1. The structure of rack with modular cells.

2.2. Notations

In this paper, the following notations are used.

n	total expected storage volume, $n = \sum n_i$
n_i	storage volume of each product to be stored in the system, $i = 1, \ldots, k$
h_i, l_i, w_i	height, length and width of load type i
H, L, W	height, length and width of an AS/RS, respectively
H_m, L_m, W_m	height, length and width of modular cell, respectively
H_f, L_f, W_f	height, length and width of cell in existing systems, respectively
N_a, N_l, N_b	number of aisles, levels and bays of an AS/RS with modular cells
N, N_1, N_2	number of aisles, levels and bays in existing AS/RS, respectively
M_i	number of cell occupied by a load type i
q_i	ratio of n_i to n, $q_i = n_i/n$
B	length of load-arm (bracket) in the rack

$\lceil x \rceil$ defined as least integer greater than or equal to x (ceiling notation)
$\lfloor x \rfloor$ defined as greatest integer less that or equal to x (floor notation)
c_1, c_2 construction cost per modular cell and equally sized cell, respectively

3. Optimal design of rack with modular cell

3.1. Mathematical model

On the basis of given parameters and variables, a mixed integer non-linear programming (non-linear MIP) model is developed for minimizing total lost spaces considering initial investment cost subject to several constraints. The objective function is as follows.

$$\begin{aligned}\text{Min TLS}_\text{m} = \sum_{i=1}^{k} nq_i(M_iH_\text{m} - h_i) \\ + 2N(H - N_1H_\text{m}) \\ + 2N\{L - N_\text{b}(L_\text{m} + 2b)\} \\ + 4bN_\text{a}N_\text{b},\end{aligned} \tag{1}$$

s.t.

$$\text{Min } h_i \leqslant H_\text{m} \leqslant \text{Max } h_i, \tag{2}$$

$$\sum_{i=1}^{p} M_i nq_i \leqslant 2N_\text{a}N_\text{b}N_1 \quad \text{and} \quad c_1N_\text{a}N_\text{b}N_1 \leqslant c_2NN_1N_2, \tag{3}$$

$$\sum q_i = 1, \tag{4}$$

$$(W_\text{m}, L_\text{m}) = \begin{cases} (\max w_i, \max l_i) & \text{if } N_\text{a}^w N_\text{b}^l \geqslant N_\text{a}^l N_\text{b}^w, \\ (\max l_i, \max w_i), & \text{otherwise,} \end{cases} \tag{5}$$

$$M_i = \left\lceil \frac{h_i}{H_\text{m}} \right\rceil, \quad M_i \geqslant 1, \tag{6}$$

$$M_i, N_\text{a}, N_\text{b}, N_1, N_\text{a}^w, N_\text{a}^l, N_\text{b}^w, N_\text{b}^l > 0, \text{ Integer,}$$

where

$$N_\text{a} = \left\lfloor \frac{W}{3W_\text{m}} \right\rfloor, \quad N_1 = \left\lfloor \frac{H}{H_\text{m}} \right\rfloor, \quad N_\text{b} = \left\lfloor \frac{L}{L_\text{m} + 2b} \right\rfloor,$$

$$N = \left\lfloor \frac{W}{3W_\text{f}} \right\rfloor, \quad N_1 = \left\lfloor \frac{H}{H_\text{f}} \right\rfloor, \quad N_2 = \left\lfloor \frac{L}{L_\text{f}} \right\rfloor$$

$$N_\text{a}^w = \left\lfloor \frac{W}{\max\ 3w_i} \right\rfloor, \quad N_\text{a}^l = \left\lfloor \frac{W}{\max\ 3l_i} \right\rfloor,$$
$$N_\text{b}^w = \left\lfloor \frac{L}{\max\ w_i} \right\rfloor, \quad N_\text{b}^l = \left\lfloor \frac{L}{\max\ l_i} \right\rfloor.$$

The objective function minimizes lost spaces within each cell, in the rack height and rack length. Constraint (2) specifies the physical bound of modular cell height. Constraint (3) is a limitation for total storage volume and cost required. Eq. (4) is a constraint on the storage volume of each product type i to be stored in the system. Eq. (5) specifies the width and length of modular cells to maximize storage volume within given area physically. Constraint (6) indicates the number of cell required for the storage of a load size h_i. Especially, the size of modular cell in height, H_m, is affected by construction cost per cell. The total lost space is in inverse proportion to the construction cost over H_m.

On the other hand, in case the different sized loads are stored in existing AS/RS, total lost space is calculated by Eq. (7). In this case, it is assumed that the height of equally sized cell, H_f, is greater than or equal to max h_i.

$$\begin{aligned}\text{Min TLS}_\text{f} = \sum_{i=1}^{k} nq_iL_\text{f}(H_\text{f} - h_i) \\ + 2N(H - N_1H_\text{f}) \\ + 2N(L - N_\text{b}L_\text{f}).\end{aligned} \tag{7}$$

3.2. Algorithm

The proposed mathematical model above is a non-linear mixed integer-programming problem (non-linear MIP). As we know, integer programs belong to a class of problems known as NP-hard. This means that there is no known algorithm for solving these problems such that the computational effort at worst increases as a polynomial in the problem size. That is, we could formulate the optimization model such as the proposed model above and also the problem could be solved theoretically by using optimization tools such as GAMS, LINGO, C-PLEX, etc. But because it is a NP-hard problem, an heuristic algorithm was suggested in order to solve the problem more effectively. Even though we could obtain the optimal solution by investing so much time and resources, it does not have any special advantage and meaning in practice.

Basically, this problem is to find a best solution, which minimizes objective function among the feasible alternatives satisfying constraints. If the required storage volume of product is larger than the total storage volume of rack configuration ($\sum M_i n q_i > 2NN_1N_2$), or if it does not satisfy cost constraint, these alternatives of rack configuration will be removed from the feasible alternative list. In order to solve effectively the model in this paper, heuristic algorithm is suggested as follows. Fig. 2 shows the procedure for deciding modular cell size.

Algorithm for the decision of modular cell size

Step 1: Decide the value of W_m and L_m by Eqs. (3)–(6) and known product sizes. Go to step 2.

Step 2: Determine the initial value (α) of H_m and incremental step size, β. Let $H_m = \alpha$ and go to step 3.

Step 3: Calculate the parameter of M_i and rack dimension parameters, N, N_1, N_2, N_a, N_l, N_b.

Step 4: Check the feasibility of the rack configuration created by $H_m = \alpha$. If it satisfies feasibility constraints, add it to a feasible alternative list and go to step 5. Otherwise, let $H_m = H_m + \beta$ and go to step 3. Repeat step 4, until the height of modular cells becomes less than or equal to max h_i, $H_m \leqslant \max h_i$.

Step 5: Select a best alternative, TLS_{m^*}, in the feasible alternative list. The best solution can also be altered among feasible alternatives by decision-makers.

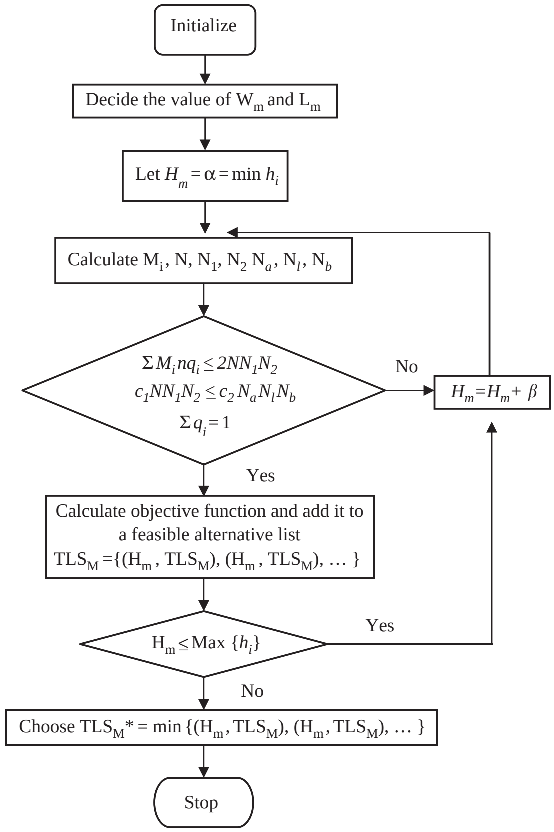

Fig. 2. The procedure for deciding modular cell size.

4. Numerical studies

In this section, the effects of proposed model are investigated through numerical examples and their results are compared to that of previous model. It is also illustrated that how the objective function behaves for decision variables. In particular, the effect of the change of modular cell height and q_i on TLS_m is investigated. Space utilization as the measure of effectiveness for the proposed model is used and it is calculated by $(TLS_f - TLS_m)/TLS_f$. The increase of space utilization will improve the storage capacity and flexibility of the warehouse system. Thus, the increase of space utilization will come to the decrease of cost in terms of system construction and operation.

The best solution of modular cell height is determined such that the total lost space over the H_m is minimized with satisfying storage volume required for the all type of loads and cost constraint. Tables 1 and 2 show the test data for three examples and the summary of solutions, respectively.

Table 1
Test data for examples

i	Ex. #1		Ex. #2		Ex. #3		l_i	w_i	h_i
	q_i	nq_i	q_i	nq_i	q_i	nq_i			
1	10	3150	10	3150	20	6300	0.8	0.4	0.3
2	15	4725	10	3150	20	6300	0.9	0.6	0.5
3	20	6300	10	3150	20	6300	0.7	0.7	0.7
4	25	7875	10	3150	20	6300	1.0	0.8	0.8
5	30	9450	60	18900	20	6300	0.8	0.9	1.2

$n = 31500$, $L = 90$, $H = 20$, $W = 25$, $H_f = 1.2$, $L_f = W_f = 1.0$, $b = 0.05$, $c_1 = 5$, $c_2 = 50$.

Table 2
The comparison of optimal solution between proposed model and existing model

Models	Parameters	Cell size L_m, W_m, H_m	Rack dimension N_a, N_l, N_b	Lost space TLS_m	Space utilization
Proposed model	CC	0.9, 1.0, 0.45	10, 44, 100	9690	25.4%
	NCC	0.9, 1.0, 0.2	10, 100, 100	5417	58.3%
Existing model		1.0, 1.0, 1.2	10, 16, 100	12995	

CC: Considering cost; NCC: Not considering cost.

The number of alternatives satisfying the feasible condition is limited by H_m. It is also affected by the value of q_i for each load type i. The performance of proposed model for space utilization is far better than that of existing model as shown in Table 2. It is increased as much as 25.4% in terms of space utilization when construction cost is considered. It means that the proposed model can store 119 unit loads more than the model of equally sized cells. This value, also, can be interpreted as the increase of shadow price or the reduction of risk for the storage demands of unexpected customers. As the rack size in height, H_m, is decided based on the required storage volume of each product type and size, the lost space is always less than that of present model which is considered in equally sized rack configuration under the unit load concept. It means that the space utilization is improved in every case; there is only the difference of large or small.

Fig. 3 shows the relationship between lost space and construction cost according to the change of H_m. As shown in Fig. 3, the construction cost can also be reduced considerably in comparison with the model of equally sized cells, $H_f = 1.2$. As stated above, the optimal size of modular cells is affected by the storage volume of each product type i, rack configuration and construction cost per cells.

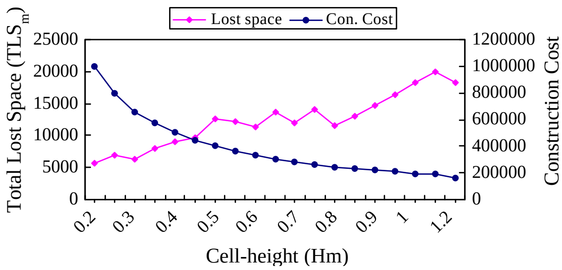

Fig. 3. The relationship between TLS_m and Cost.

In addition to these advantages, the performance evaluation of proposed system could be easily estimated by existing model according to the storage policy. That is, the rack structure with modular cells can accommodate all types of storage policies that can be changed according to storage demands. Consequently, all of these advantages and effectiveness are entirely derived from the structural changes of rack of an AS/RS.

5. Conclusions

We have proposed the model of AS/RS with the rack of modular cells that can hold different size of unit loads. The mathematical model of cell size was also developed considering storage volume for each load type. Comparison between the previous model with the rack of equally sized cells and the proposed new model shows much of difference in terms of capability for coping with a business environment is changing. To the best of our knowledge, an AS/RS with the rack of modular cells is studied first in this paper.

The proposed model in this paper has clearly much advantages compared to the model that has been researched until now. It, of course, there may be some limitations in practice, such as technical problems for designing and constructing to deal with heavy weight loads. However, the new model of AS/RS with modular cells proposed in this paper, under the concept of non-unit-load size, could be a very useful alternative for meeting the limitations and/or problems of existing systems and the business environment that is rapidly changing. AS/RS to be built henceforth should be able to provide flexibility to variable customer's needs.

References

Ashayeri, J., Gelders, L.F., Van Wassenhove, L., 1985. A microcomputer-based optimization model for the design of automated warehouses. International Journal of Production Research 23 (4), 825–839.

Azadivar, F., 1989. Optimum allocation of resources between the random access and rack storage spaces in an automated warehousing system. International Journal of Production Research 27 (1), 119–131.

Bafna, K.M., Reed Jr., R., 1972. An analytical approach to design of high-rise stacker crane warehouse systems. Journal of Industrial Engineering 8, 8–14.

Graves, S.C., Hausman, W.H., Schwarz, L.B., 1977. Storage–retrieval interleaving in automatic warehousing systems. Management Science 23 (9), 935–945.

Groover, M.P., 1987. Automation, production systems, and computer-integrated manufacturing. Prentice-Hall, Englewood Cliffs, NJ.

Han, M.H., McGinnis, L.F., Shieh, J.S., White, J.A., 1987. On sequencing retrievals in an automated storage/retrieval system. IIE Transactions 19 (1), 56–66.

Karasawa, Y., Nakayama, H., Dohi, S., 1980. Trade-off analysis for optimal design of automated warehouses. International Journal of Systems Science 11 (5), 567–576.

Lee, H.F., 1997. Performance analysis for automated storage and retrieval systems. IIE Transactions 29 (1), 15–28.

Lee, Y.H., Tanchoco, J.M.A., Chun, S.J., 1999. Performance estimation models for AS/RS with unequal sized cells. International Journal of Production Research 37 (18), 4197–4216.

Perry, R.F., Hoover, S.V., Freeman, D.R., 1983. Design of an automated storage/retrieval system using simulation modeling. Institute of Industrial Engineering, ICAW Proceedings, Atlanta, Georgia, pp. 57–63.

Perry, R.F., Hoover, S.F., Freeman, D.R., 1984. An optimum-seeking approach to the design of automated storage/retrieval systems. Proceedings of the 1984 Winter Simulation Conference pp. 349–354.

Roberts, S.D., Reed Jr., R., 1972. Optimal warehouse bay configurations. IIE Transactions 4 (3), 178–185.

Rosenblatt, M.J., Roll, Y., 1984. Warehouse design with storage policy considerations. International Journal of Production Research 22 (5), 809–821.

Rosenblatt, M.J., Eynan, A., 1989. Deriving the optimal boundaries for class-based automatic storage/retrieval systems. Management Science 35 (12), 1519–1524.

Rosenblatt, M.J., Roll, Y., Zyser, V., 1993. A combined optimization and simulation approach for designing automated storage/retrieval systems. IIE Transactions 25 (1), 40–50.