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Optimal design of hydraulic support

M. Oblak, B. Harl and B. Butinar

Abstract This paper describes a procedure for optimal determination of two groups of parameters of a hydraulic support employed in the mining industry. The procedure is based on mathematical programming methods. In the first step, the optimal values of some parameters of the leading four-bar mechanism are found in order to ensure the desired motion of the support with minimal transversal displacements. In the second step, maximal tolerances of the optimal values of the leading four-bar mechanism are calculated, so the response of hydraulic support will be satisfying.

Key words four-bar mechanism, optimal design, mathematical programming, approximation method, tolerance

1 Introduction

The designer aims to find the best design for the mechanical system considered. Part of this effort is the optimal choice of some selected parameters of a system. Methods of mathematical programming can be used, if a suitable mathematical model of the system is made. Of course, it depends on the type of the system. With this formulation, good computer support is assured to look for optimal parameters of the system.

The hydraulic support (Fig. 1) described by Harl (1998) is a part of the mining industry equipment in the mine Velenje-Slovenia, used for protection of working environment in the gallery. It consists of two four-bar mechanisms FEDG and AEDB as shown in Fig. 2. The mechanism AEDB defines the path of coupler point C and the mechanism FEDG is used to drive the support by a hydraulic actuator.

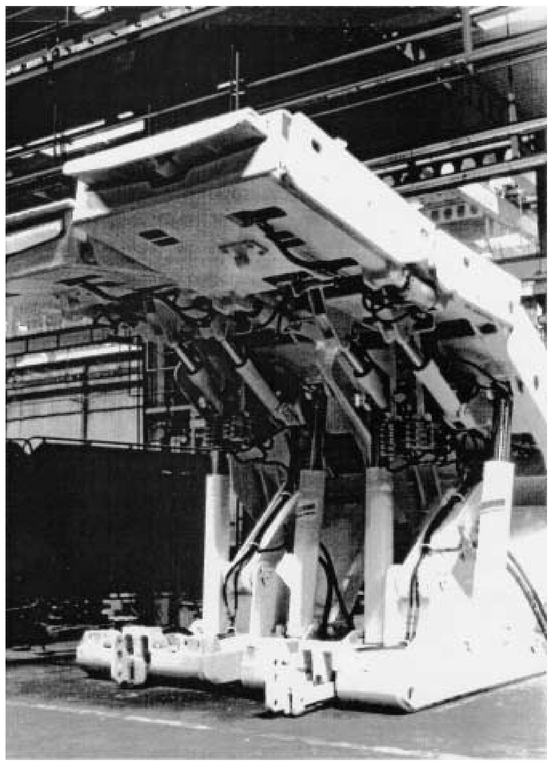

Fig. 1 Hydraulic support

It is required that the motion of the support, more precisely, the motion of point C in Fig. 2, is vertical with minimal transversal displacements. If this is not the case, the hydraulic support will not work properly because it is stranded on removal of the earth machine.

A prototype of the hydraulic support was tested in a laboratory (Grm 1992). The support exhibited large transversal displacements, which would reduce its employability. Therefore, a redesign was necessary. The project should be improved with minimal cost if pos-

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M. Oblak[1], B. Harl[2] and B. Butinar[3]

[1] Faculty of Mechanical Engineering, Smetanova 17, 2000 Maribor, Slovenia
e-mail: maks.oblak@uni-mb.si
[2] M.P.P. Razvoj d.o.o., Ptujska 184, 2000 Maribor, Slovenia
e-mail: bostjan.harl@uni-mb.si
[3] Faculty of Chemistry and Chemical Engineering, Smetanova 17, 2000 Maribor, Slovenia
e-mail: branko.butinar@uni-mb.si

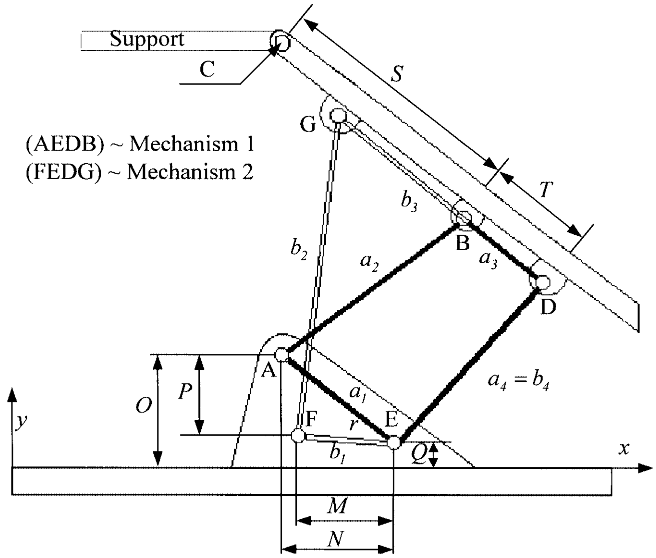

Fig. 2 Two four-bar mechanisms

sible. It was decided to find the best values for the most problematic parameters a_1, a_2, a_4 of the leading four-bar mechanism AEDB with methods of mathematical programming. Otherwise it would be necessary to change the project, at least mechanism AEDB.

The solution of above problem will give us the response of hydraulic support for the ideal system. Real response will be different because of tolerances of various parameters of the system, which is why the maximal allowed tolerances of parameters a_1, a_2, a_4 will be calculated, with help of methods of mathematical programming.

2
The deterministic model of the hydraulic support

At first it is necessary to develop an appropriate mechanical model of the hydraulic support. It could be based on the following assumptions:

- the links are rigid bodies,
- the motion of individual links is relatively slow.

The hydraulic support is a mechanism with one degree of freedom. Its kinematics can be modelled with synchronous motion of two four-bar mechanisms FEDG and AEDB (Oblak *et al.* 1998). The leading four-bar mechanism AEDB has a decisive influence on the motion of the hydraulic support. Mechanism 2 is used to drive the support by a hydraulic actuator. The motion of the support is well described by the trajectory $\mathcal{L}$ of the coupler point C. Therefore, the task is to find the optimal values of link lengths of mechanism 1 by requiring that the trajectory of the point C is as near as possible to the desired trajectory $\mathcal{K}$.

The synthesis of the four-bar mechanism 1 has been performed with help of kinematics equations of motion given by Rao and Dukkipati (1989). The general situation is depicted in Fig. 3.

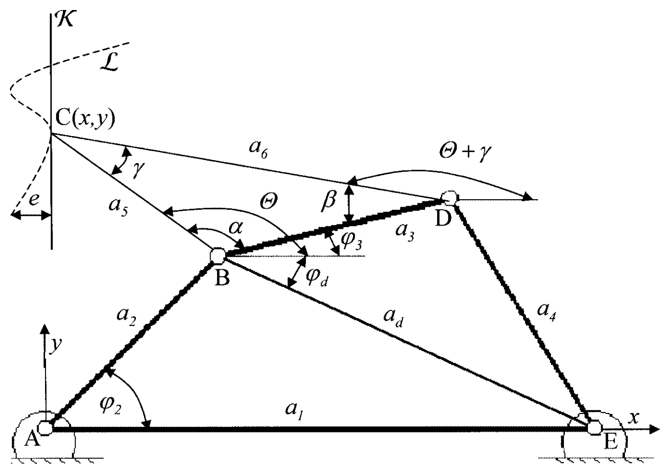

Fig. 3 Trajectory $\mathcal{L}$ of the point C

Equations of trajectory $\mathcal{L}$ of the point C will be written in the coordinate frame considered. Coordinates x and y of the point C will be written with the typical parameters of a four-bar mechanism $a_1, a_2, \ldots, a_6$. The coordinates of points B and D are

$$x_B = x - a_5 \cos\Theta\,, \tag{1}$$

$$y_B = y - a_5 \sin\Theta\,, \tag{2}$$

$$x_D = x - a_6 \cos(\Theta+\gamma)\,, \tag{3}$$

$$y_D = y - a_6 \sin(\Theta+\gamma)\,. \tag{4}$$

The parameters $a_1, a_2, \ldots, a_6$ are related to each other by

$$x_B^2 + y_B^2 = a_2^2\,, \tag{5}$$

$$(x_D - a_1)^2 + y_D^2 = a_4^2\,. \tag{6}$$

By substituting (1)–(4) into (5)–(6) the response equations of the support are obtained as

$$(x - a_5\cos\Theta)^2 + (y - a_5\sin\Theta)^2 - a_2^2 = 0\,, \tag{7}$$

$$[x - a_6\cos(\Theta+\gamma) - a_1]^2 +$$

$$[y - a_6\sin(\Theta+\gamma)]^2 - a_4^2 = 0\,. \tag{8}$$

This equation represents the base of the mathematical model for calculating the optimal values of parameters a_1, a_2, a_4.

2.1 Mathematical model

The mathematical model of the system will be formulated in the form proposed by Haug and Arora (1979):

$$\min\ f(\mathbf{u},\mathbf{v})\,, \tag{9}$$

subject to constraints

$$g_i(\mathbf{u},\mathbf{v}) \le 0\,, \quad i=1,2,\dots,\ell\,, \tag{10}$$

and response equations

$$h_j(\mathbf{u},\mathbf{v}) = 0\,, \quad j=1,2,\dots,m\,. \tag{11}$$

The vector $\mathbf{u}=[u_1\ \dots u_n]^T$ is called the vector of design variables, $\mathbf{v}=[v_1\ \dots v_m]^T$ is the vector of response variables and f in (9) is the objective function.

To perform the optimal design of the leading four-bar mechanism AEDB, the vector of design variables is defined as

$$\mathbf{u}=[a_1\quad a_2\quad a_4]^T\,, \tag{12}$$

and the vector of response variables as

$$\mathbf{v}=[x\quad y]^T\,. \tag{13}$$

The dimensions a_3, a_5, a_6 of the corresponding links are kept fixed.

The objective function is defined as some "measure of difference" between the trajectory $\mathcal{L}$ and the desired trajectory $\mathcal{K}$ as

$$f(\mathbf{u},\mathbf{v}) = \max\,[g_0(y)-f_0(y)]^2\,, \tag{14}$$

where $x=g_0(y)$ is the equation of the curve $\mathcal{K}$ and $x=f_0(y)$ is the equation of the curve $\mathcal{L}$.

Suitable limitations for our system will be chosen. The system must satisfy the well-known Grasshoff conditions

$$(a_3+a_4)-(a_1+a_2) \le 0\,, \tag{15}$$

$$(a_2+a_3)-(a_1+a_4) \le 0\,. \tag{16}$$

Inequalities (15) and (16) express the property of a four-bar mechanism, where the links a_2, a_4 may only oscillate.

The condition

$$\underline{\mathbf{u}} \le \mathbf{u} \le \overline{\mathbf{u}} \tag{17}$$

prescribes the lower and upper bounds of the design variables.

The problem (9)–(11) is not directly solvable with the usual gradient-based optimization methods. This could be circumvented by introducing an artificial design variable u_{n+1} as proposed by Hsieh and Arora (1984). The new formulation exhibiting a more convenient form may be written as

$$\min\ u_{n+1}\,, \tag{18}$$

subject to

$$g_i(\mathbf{u},\mathbf{v}) \le 0\,, \quad i=1,2,\dots,\ell\,, \tag{19}$$

$$f(\mathbf{u},\mathbf{v}) - u_{n+1} \le 0\,, \tag{20}$$

and response equations

$$h_j(\mathbf{u},\mathbf{v}) = 0\,, \quad j=1,2,\dots,m\,, \tag{21}$$

where $\mathbf{u}=[u_1\ \dots u_n\ u_{n+1}]^T$ and $\mathbf{v}=[v_1\ \dots v_m]^T$.

A nonlinear programming problem of the leading four-bar mechanism AEDB can therefore be defined as

$$\min\ a_7\,, \tag{22}$$

subject to constraints

$$(a_3+a_4)-(a_1+a_2) \le 0\,, \tag{23}$$

$$(a_2+a_3)-(a_1+a_4) \le 0\,, \tag{24}$$

$$\underline{a_1} \le a_1 \le \overline{a_1}\,, \quad \underline{a_2} \le a_2 \le \overline{a_2}\,,$$

$$\underline{a_4} \le a_4 \le \overline{a_4}\,, \tag{25}$$

$$[g_0(y)-f_0(y)]^2 - a_7 \le 0\,, \quad (y \in |\underline{y},\overline{y}|)\,, \tag{26}$$

and response equations

$$(x-a_5\cos\Theta)^2 + (y-a_5\sin\Theta)^2 - a_2^2 = 0\,, \tag{27}$$

$$[x-a_6\cos(\Theta+\gamma)-a_1]^2 +$$

$$[y-a_6\sin(\Theta+\gamma)]^2 - a_4^2 = 0\,. \tag{28}$$

This formulation enables the minimization of the difference between the transversal displacement of the point C and the prescribed trajectory $\mathcal{K}$. The result is the optimal values of the parameters a_1, a_2, a_4.

3 The stochastic model of the hydraulic support

The mathematical model (22)–(28) may be used to calculate such values of the parameters a_1, a_2, a_4, that the "difference between trajectories $\mathcal{L}$ and $\mathcal{K}$" is minimal. However, the real trajectory $\mathcal{L}$ of the point C could deviate from the calculated values because of different influences. The suitable mathematical model deviation could be treated dependently on tolerances of parameters a_1, a_2, a_4.

The response equations (27)–(28) allow us to calculate the vector of response variables $\mathbf{v}$ in dependence on the vector of design variables $\mathbf{u}$. This implies $\mathbf{v} = \tilde{\mathbf{h}}(\mathbf{u})$. The function $\tilde{\mathbf{h}}$ is the base of the mathematical model (22)–(28), because it represents the relationship between the vector of design variables $\mathbf{u}$ and response $\mathbf{v}$ of our mechanical system. The same function $\tilde{\mathbf{h}}$ can be used to calculate the maximal allowed values of the tolerances Δa_1, Δa_2, Δa_4 of parameters a_1, a_2, a_4.

In the stochastic model the vector $\mathbf{u} = [u_1 \ldots u_n]^T$ of design variables is treated as a random vector $\mathbf{U} = [U_1 \ldots U_n]^T$, meaning that the vector $\mathbf{v} = [v_1 \ldots v_m]^T$ of response variables is also a random vector $\mathbf{V} = [V_1 \ldots V_m]^T$,

$$\mathbf{V} = \tilde{\mathbf{h}}(\mathbf{U}) \,. \tag{29}$$

It is supposed that the design variables $U_1, \ldots, U_n$ are independent from the probability point of view and that they exhibit normal distribution, $U_k \sim N(\mu_k, \sigma_k)$ $(k = 1, 2, \ldots, n)$. The main parameters μ_k and σ_k $(k = 1, 2, \ldots, n)$ could be bound with technological notions such as nominal measures, $\mu_k = u_k$ and tolerances, e.g. $\Delta u_k = 3\sigma_k$, so events

$$\mu_k - \Delta u_k \le U_k \le \mu_k + \Delta u_k \,, \quad k = 1, 2, \ldots, n \,, \tag{30}$$

will occur with the chosen probability.

The probability distribution function of the random vector $\mathbf{V}$, that is searched for depends on the probability distribution function of the random vector $\mathbf{U}$ and it is practically impossible to calculate. Therefore, the random vector $\mathbf{V}$ will be described with help of "numbers characteristics", that can be estimated by Taylor approximation of the function $\tilde{\mathbf{h}}$ in the point $\mathbf{u} = [u_1 \ldots u_n]^T$ or with help of the Monte Carlo method in the papers by Oblak (1982) and Harl (1998).

3.1 The mathematical model

The mathematical model for calculating optimal tolerances of the hydraulic support will be formulated as a nonlinear programming problem with independent variables

$$\mathbf{w} = [\Delta a_1 \quad \Delta a_2 \quad \Delta a_4]^T \,, \tag{31}$$

and objective function

$$f(\mathbf{w}) = \frac{1}{\Delta a_1} + \frac{1}{\Delta a_2} + \frac{1}{\Delta a_4} \tag{32}$$

with conditions

$$\sigma_Y - E \le 0 \,, \tag{33}$$

$$\underline{\Delta a_1} \le \Delta a_1 \le \overline{\Delta a_1} \,, \quad \underline{\Delta a_2} \le \Delta a_2 \le \overline{\Delta a_2} \,,$$

$$\underline{\Delta a_4} \le \Delta a_4 \le \overline{\Delta a_4} \,. \tag{34}$$

In (33) E is the maximal allowed standard deviation σ_Y of coordinate x of the point C and

$$\sigma_Y = \frac{1}{\sqrt{6}} \sqrt{\sum_{j \in \mathcal{A}} \left(\frac{\partial g_1}{\partial a_j} (\mu_1, \mu_2, \mu_4) \right)^2 \Delta a_j} \,,$$

$$\mathcal{A} = \{1, 2, 4\} \,. \tag{35}$$

The nonlinear programming problem for calculating the optimal tolerances could be therefore defined as

$$\min \left(\frac{1}{\Delta a_1} + \frac{1}{\Delta a_2} + \frac{1}{\Delta a_4} \right) \,, \tag{36}$$

subject to constraints

$$\sigma_Y - E \le 0 \,, \tag{37}$$

$$\underline{\Delta a_1} \le \Delta a_1 \le \overline{\Delta a_1} \,, \quad \underline{\Delta a_2} \le \Delta a_2 \le \overline{\Delta a_2} \,,$$

$$\underline{\Delta a_4} \le \Delta a_4 \le \overline{\Delta a_4} \,. \tag{38}$$

4 Numerical example

The carrying capability of the hydraulic support is 1600 kN. Both four-bar mechanisms AEDB and FEDG must fulfill the following demand:

- they must allow minimal transversal displacements of the point C, and
- they must provide sufficient side stability.

The parameters of the hydraulic support (Fig. 2) are given in Table 1.

The drive mechanism FEDG is specified by the vector

$$[b_1, b_2, b_3, b_4]^T = [400, (1325 + d), 1251, 1310]^T \text{ (mm)} \,, \tag{39}$$

and the mechanism AEDB by

$$[a_1, a_2, a_3, a_4]^T = [674, 1360, 382, 1310]^T \text{ (mm)} \,. \tag{40}$$

In (39), the parameter d is a walk of the support with maximal value of 925 mm. Parameters for the shaft of the mechanism AEDB are given in Table 2.

Table 1 Parameters of hydraulic support

Sign	Length (mm)
M	110
N	510
O	640
P	430
Q	200
S	1415
T	380

Table 2 Parameters of the shaft for mechanism AEDB

Sign	
a_5	1427.70 mm
a_6	1809.68 mm
α	179.34°
β	0.52°
γ	0.14°

4.1 Optimal links of mechanism AEDB

With this data the mathematical model of the four-bar mechanisms AEDB could be written in the form of (22)–(28). A straight line is defined by $x = 65$ (mm) (Fig. 3) for the desired trajectory of the point C. That is why condition (26) is

$$(x - 65) - a_7 \leq 0\,. \tag{41}$$

The angle between links AB and AE may vary between 76.8° and 94.8°. The condition (41) will be discretized by taking into account only the points $x_1, x_2, \ldots, x_{19}$ in Table 3. These points correspond to the angles φ_{2_1}, $\varphi_{2_2}, \ldots, \varphi_{2_{19}}$ of the interval $[76.8°, 94.8°]$ at regular intervals of 1°.

The lower and upper bounds of design variables are

$$\underline{\mathbf{u}} = [640, 1330, 1280, 0]^T \text{ (mm)}, \tag{42}$$

$$\overline{\mathbf{u}} = [700, 1390, 1340, 30]^T \text{ (mm)}. \tag{43}$$

The nonlinear programming problem is formulated in the form of (22)–(28). The problem is solved by the optimizer described by Kegl *et al.* (1991) based on approximation method. The design derivatives are calculated numerically by using the direct differentiation method.

The starting values of design variables are

$$\left[{}^0a_1, {}^0a_2, {}^0a_4, {}^0a_7\right]^T = [674, 1360, 1310, 30]^T \text{ (mm)}. \tag{44}$$

The optimal design parameters after 25 iterations are

$$\mathbf{u}^* = [676.42, 1360.74, 1309.88, 3.65]^T \text{ (mm)}. \tag{45}$$

In Table 3 the coordinates x and y of the coupler point C are listed for the starting and optimal designs, respectively.

Table 3 Coordinates x and y of the point C

Angle φ_2(°)	x_{start} (mm)	y_{start} (mm)	x_{end} (mm)	y_{end} (mm)
76.8	66.78	1784.87	69.47	1787.50
77.8	65.91	1817.67	68.74	1820.40
78.8	64.95	1850.09	67.93	1852.92
79.8	63.92	1882.15	67.04	1885.07
80.8	62.84	1913.85	66.12	1916.87
81.8	61.75	1945.20	65.20	1948.32
82.8	60.67	1976.22	64.29	1979.44
83.8	59.65	2006.91	63.46	2010.23
84.8	58.72	2037.28	62.72	2040.70
85.8	57.92	2067.35	62.13	2070.87
86.8	57.30	2097.11	61.73	2100.74
87.8	56.91	2126.59	61.57	2130.32
88.8	56.81	2155.80	61.72	2159.63
89.8	57.06	2184.74	62.24	2188.67
90.8	57.73	2213.42	63.21	2217.46
91.8	58.91	2241.87	64.71	2246.01
92.8	60.71	2270.08	66.85	2274.33
93.8	63.21	2298.09	69.73	2302.44
94.8	66.56	2325.89	70.50	2330.36

Figure 4 illustrates the trajectories $\mathcal{L}$ of the point C for the starting (hatched) and optimal (full) design as well as the straight line $\mathcal{K}$.

4.2 Optimal tolerances for mechanism AEDB

In the nonlinear programming problem (36)–(38), the chosen lower and upper bounds of independent variables Δa_1, Δa_2, Δa_4 are

$$\underline{\mathbf{w}} = [0.001, 0.001, 0.001]^T \text{ (mm)}, \tag{46}$$

$$\overline{\mathbf{w}} = [3.0, 3.0, 3.0]^T \text{ (mm)}. \tag{47}$$

The starting values of the independent variables are

$$\mathbf{w}_0 = [0.1, 0.1, 0.1]^T \text{ (mm)}\,. \tag{48}$$

The allowed deviation of the trajectory was chosen for two cases as $E = 0.01$ and $E = 0.05$. In the first case, the

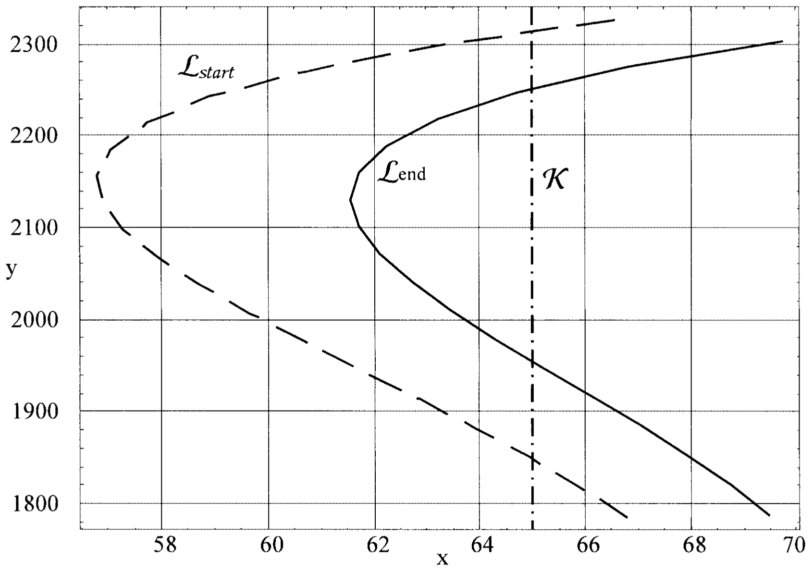

Fig. 4 Trajectories of the point C

Table 4 Optimal tolerances for $E = 0.01$

Sign	Value (mm)
Δa_1	0.01917
Δa_2	0.00868
Δa_4	0.00933

Table 5 Optimal tolerances for $E = 0.05$

Sign	Value (mm)
Δa_1	0.09855
Δa_2	0.04339
Δa_4	0.04667

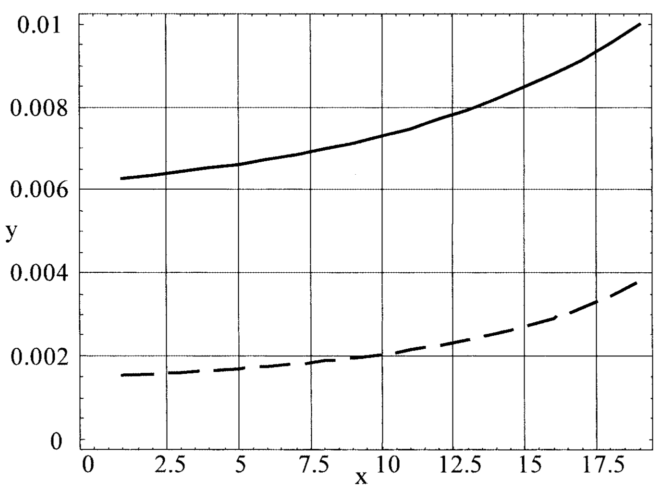

Fig. 5 Standard deviations for $E = 0.01$

optimal tolerances for the design variables a_1, a_2, a_4 were calculated after 9 iterations. For $E = 0.05$ the optimum was obtained after 7 iterations. The results are given in Tables 4 and 5.

In Figs. 5 and 6 the standard deviations are calculated by the Monte Carlo method and with Taylor approximation (full line represented Taylor approximation), respectively.

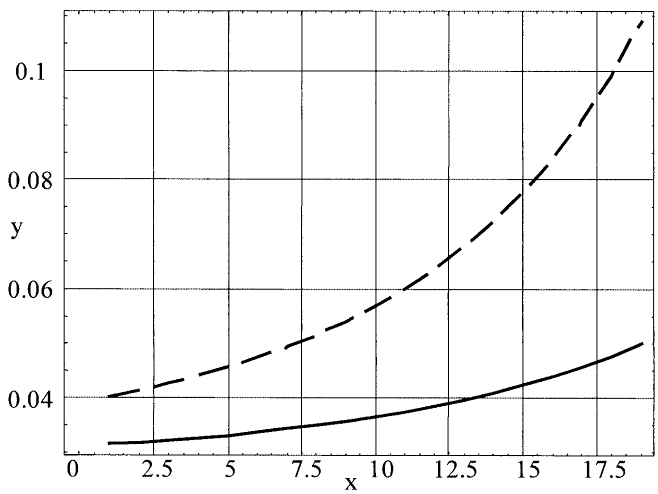

Fig. 6 Standard deviations for $E = 0.05$

5 Conclusions

With a suitable mathematical model of the system and by employing mathematical programming, the design of the hydraulic support was improved, and better performance was achieved. However, due to the results of optimal tolerances, it might be reasonable to take into consideration a new construction. This is especially true for the mechanism AEDB, since very small tolerances raise the costs of production.

References

Grm, V. 1992: *Optimal synthesis of four-bar mechanism*. MSc. Thesis. Faculty of Mechanical Engineering Maribor

Harl, B. 1998: *Stochastic analyses of hydraulic support 2S*. MSc. Thesis. Faculty of Mechanical Engineering Maribor

Haug, E.J.; Arora, J.S. 1979: *Applied optimal design*. New York: Wiley

Hsieh, C.; Arora, J. 1984: Design sensitivity analysis and optimisation of dynamic response. *Comp. Meth. Appl. Mech. Engrg.* **43**, 195–219

Kegl, M.; Butinar, B.; Oblak, M. 1991: Optimization of mechanical systems: On strategy of non-linear first-order approximation. *Int. J. Numer. Meth. Eng.* **33**, 223–234

Oblak, M. 1982: Numerical analyses of structures part II. Faculty of Mechanical Engineering Maribor

Oblak, M.; Ciglarič, I.; Harl, B. 1998: The optimal synthesis of hydraulic support. *ZAMM* **3**, 1027–1028

Rao, S.S.; Dukkipati, R.V. 1989: *Mechanism and machine theory*. New Delhi: Wiley & Sons