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Torsional vibration analysis of lathe spindle system with unbalanced workpiece

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Abstract: For the purpose of analyzing the torsional vibration caused by the gravitational unbalance torque arisen in a spindle system when it is machining heavy work piece, a 10-DOF lumped parameter model was made for the machine tool spindle system with geared transmission. By using the elementary method and Runge-Kutta method in Matlab, the eigenvalue problem was solved and the pure torsional vibration responses were obtained and examined. The results show that the spindle system cannot operate in the desired constant rotating speed as far as the gravitational unbalance torque is engaged, so it may cause bad effect on machining accuracy. And the torsional vibration increases infinitely near the resonant frequencies, so the spindle system cannot operate normally during these spindle speed ranges.

Key words: unbalanced work piece; gravitational unbalance torque; torsional vibration; spindle system; geared transmission

1 Introduction

When a lathe is machining heavy work piece with unbalance weight such as crankshaft, its main spindle system cannot operate in the desired constant rotating speed, because unbalance torque usually arises in the work piece. In order to keep constant operation speed against the gravitational unbalance torque, usually control motor is adopted in the spindle system with geared transmission in addition to main driving motor. However, the static pre-torque produced by the control motor is half the unbalance torque, so it may compensate at most half the unbalance torque of the work piece. The resulting rotating speed variations in the main spindle or work piece may cause undesirable or bad effects on machining accuracy.

Recently, there are some remarkable researches on torsional vibration of turning lathe spindle system with geared transmission. Especially, SARAVANAN et al [1], GAO and HAO [2], and YUAN et al [3] have focused on the torsional vibration caused by unbalancing; CHEN et al [4], HSIEH et al [5], and HUANG [6] have researched on coupled torsional vibrations; LEES [7], PATEL and DARPE [8], and NEUGEBAUER et al [9] have researched on lateral vibrations; and CHOI et al [10] have researched on the geared transmission system; and there are also lots of researches on the simplified mathematical modeling of main spindle system [11−15].

However, the torsional vibration of the spindle system with unbalanced work piece was not taken into consideration in all the researches mentioned above. If the spindle system is added with an unbalanced work piece, the torsional vibration of the spindle system will become more complex, even though driving motor torque is not applied during constant speed operation.

For the purpose of analyzing the torsional vibration, a 10-DOF lumped parameter model was made for the spindle system with geared transmission of a lathe with unbalanced work piece. The torsional vibration of the spindle system was analyzed by using Matlab and the eigenvalue problem of the system was solved by using the elementary method [16−17]. Then, forced vibration responses of the spindle system were obtained under the driving torque together with gravitational unbalance torque.

By comparing the computed forced vibration responses for the two cases: the spindle system with and without unbalanced work piece, the effects on the torsional vibration responses of the spindle caused by the gravitational unbalance torque were able to be clarified. And the pure torsional velocity response of the spindle, which may be an important estimator for machining accuracy, was obtained and examined.

2 Theoretical vibration analyses

2.1 Mathematical modelling

From the schematic diagram of the main spindle system with geared transmission as shown in Fig.1, a 10-DOF mathematical model was made, as shown in

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Fig.2, where J_i represents the mass moment of inertia of the i-th equivalent rotor and k_{ij} represents the torsional spring stiffness of the shaft between the i-th and j-th equivalent rotor.

2.2 Equation of motion

The equation of motion of the system can be derived by Newton's law as

$$\boldsymbol{J}\ddot{\boldsymbol{\theta}}+\boldsymbol{K}_{\mathrm{t}}\boldsymbol{\theta}=\boldsymbol{T} \tag{1}$$

where $\boldsymbol{J}$ represents the inertia matrix of the system; $\boldsymbol{T}$ represents the input torque vector matrix of the system and $\boldsymbol{K}_{\mathrm{t}}$ represents the torsional stiffness coefficient matrix.

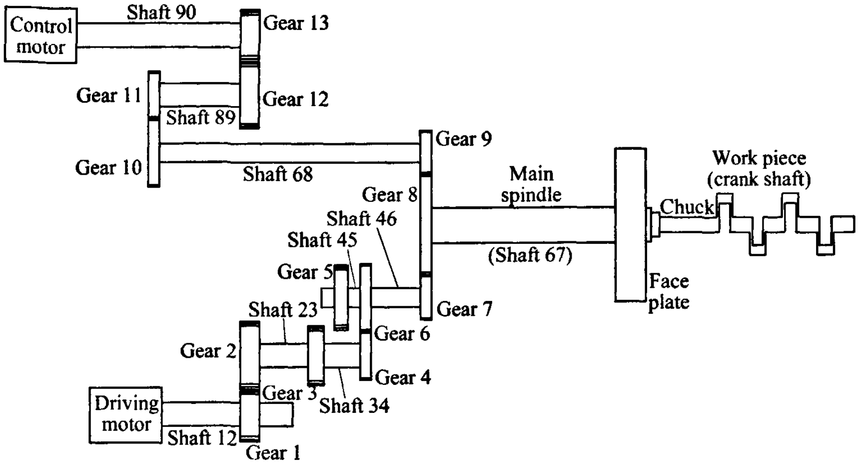

Fig.1 Schematic diagram of main spindle system

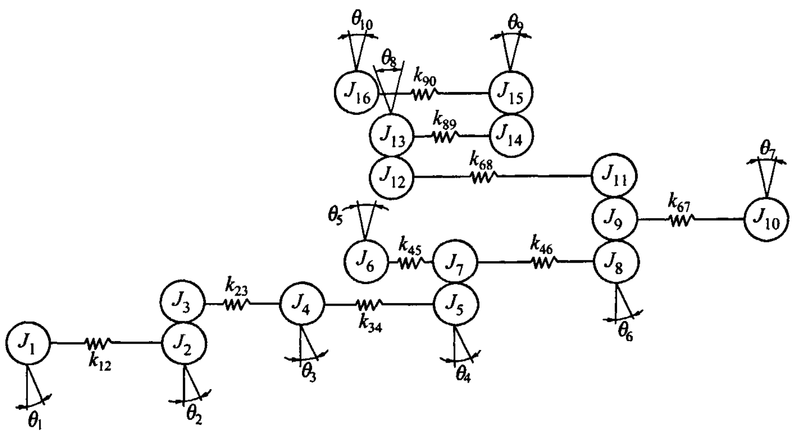

Fig.2 Mathematical modeling of main spindle system

$$\boldsymbol{J}=\begin{bmatrix} J_{\mathrm{eq1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_{\mathrm{eq2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{\mathrm{eq3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{\mathrm{eq4}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{\mathrm{eq5}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{\mathrm{eq6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{\mathrm{eq7}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{\mathrm{eq8}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{\mathrm{eq9}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{\mathrm{eq10}} \end{bmatrix} \tag{2}$$

$$T=[T_m(t)\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ T_u(t)\ \ 0\ \ 0\ \ T_c(t)]^T \tag{3}$$

$$K_t=\begin{bmatrix} k_{12} & -k_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{12} & k_{12}+k_{23}\frac{z_1}{z_2} & -k_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_{23}\frac{z_1}{z_2} & k_{23}+k_{34} & -k_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{34} & k_{34}+k_{45}\frac{z_4}{z_6}+k_{46}\frac{z_4}{z_6} & -k_{45} & -k_{46} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{45}\frac{z_4}{z_6} & k_{45} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{46} & 0 & k_{46}+k_{67}\frac{z_7}{z_8}+k_{68}\frac{z_7}{z_9} & -k_{67} & -\frac{z_{11}}{z_{10}}k_{68} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{z_7}{z_8}k_{67} & k_{67} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{z_7}{z_9}k_{68} & 0 & k_{89}+k_{68}\frac{z_{11}}{z_{10}} & -\frac{z_{13}}{z_{12}}k_{89} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{89} & k_{90}+k_{89}\frac{z_{13}}{z_{12}} & -k_{90} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{90} & k_{90} \end{bmatrix} \tag{4}$$

where J_{eqi} represents the gear ratio; $T_m(t)$ represents the driving motor torque; $T_u(t)$ represents the unbalance torque; $T_c(t)$ represents the control motor torque; z_i represents the number of teeth of the gear.

k_i is determined as

$$k_i=\frac{G_i I_i}{L_i} \tag{5}$$

where G_i represents the shear modulus of the i-th shaft, I_i represents the cross section area moment of inertia of i-th shaft and L_i represents the length of the i-th shaft.

2.3 Gravitational unbalance torque

The gravitational unbalance torque may be caused by three reasons: journal unbalance, unbalance in crankpin and misalignment of journal.

3-dimensional (3D) model of a crankshaft, which is a workpiece, is shown in Fig.3. It can be simplified as a uniform shaft with evenly phased six unbalance weights as shown in Fig.4, where m_i represents the unbalance mass of the i-th crankpin; r_i represents the radius of the i-th unbalance mass; g represents the gravitational acceleration and f_{g_i} represents the gravity of the i-th crankpin.

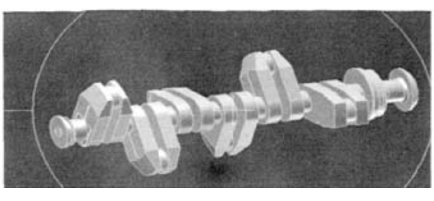

Fig.3 3D model of 6 pins crankshaft

The gravitational unbalance torque of n-crankpins, T_{up}, can be derived as

$$T_{up}=\sum_{i=1} m_i r_i g\cdot\cos(\phi_i+\theta(t)) \tag{6}$$

where ϕ_i represents the phase angle of the i-th unbalance mass.

The pure torsional angular response is defined as $\theta_\phi=\theta(t)-\theta_k$, where θ_k is the kinematic angular displacement and it is much bigger than θ_ϕ to ignore the θ_ϕ, so in the Eq.(6), $\theta(t)=\theta_k+\theta_\phi\approx\theta_k$.

Therefore, the gravitational unbalancing torque of 6 crankpins can be expressed as

$$\begin{aligned} T_{up}=&T_{up1}+T_{up2}+T_{up3}+T_{up4}+T_{up5}+T_{up6}=r_1 m_1 g\sin\theta_k- \\ &r_2 m_2 g\cos(-\phi_0+\theta_k)+r_3 m_3 g\cos(\phi_0+\theta_k)- \\ &r_4 m_4 g\cdot\sin\theta_k+r_5 m_5 g\cos(-\phi_0+\theta_k)- \\ &r_6 m_6 g\cos(\phi_0+\theta_k) \end{aligned} \tag{7}$$

where $\phi_0=\frac{\pi}{6}$ and T_{upi} represents the gravitational unbalance torque of the i-th crankpin.

For the convenience of computation, it is assumed that there is no loss of generality, the unbalance of the journals, u_j, is 1% and the error in radius of gyration of journals, e_j, is 3.5%. The resulting unbalanced torque of journals can be expressed as

$$T_{u1}(t)=m_{uj}r_{uj}g\cos\theta_k=(u_j m_j)r_{uj}g\cos\theta_k \tag{8}$$

where m_{uj} represent the unbalance mass of the journal; r_{uj} represents the radius of the unbalance mass and m_j represents the mass of the journal.

The misalignment torque of journals can be expressed as

$$T_{u2}(t)=m_j r_{mj}g\cos\theta_k=m_j(e_j d_j)g\cos\theta_k \tag{9}$$

So the resultant unbalancing torque of crankshaft in this system can be expressed as

$$T_u(t)=T_{u1}(t)+T_{u2}(t)+T_{up} \tag{10}$$

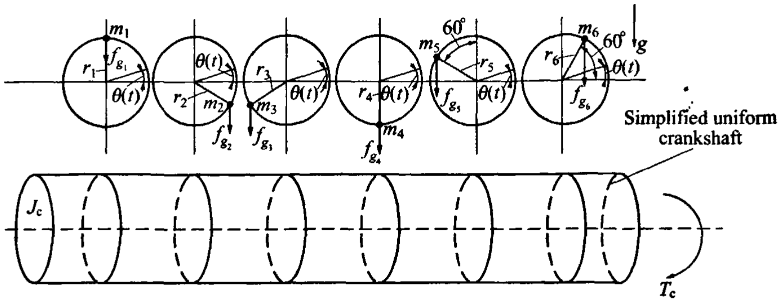

Fig.4 6-unbalance weights in simplified crankshaft

2.4 Solving equations of motion

2.4.1 Eigenvalue solution

For solving the eigenvalue problem, let $T(t)$=0 and assume that a solution is $\theta(t)=\boldsymbol{\Theta}\exp(\mathrm{i}\omega t)$, where θ represents the maximum amplitude and ω represents frequency, then the free vibration equation of the system becomes eigenvalue problem:

$$\boldsymbol{K}_t-\omega^2\boldsymbol{J}\boldsymbol{\Theta}=0 \quad (11)$$

To make the stiffness matrix symmetric, all the gear ratios in Eq.(4) are assumed unit. And J_{eq1}=2.35, J_{eq2}=1.89, J_{eq3}=1.48, J_{eq4}=1.97, J_{eq5}=4.25, J_{eq6}=86.37, J_{eq7}=3 612.4 (in Case 1), J_{eq7}=7 802.4 (in Case 2), J_{eq8}= 6.63, J_{eq9}=0.53, J_{eq10}=1.01 kg·m^2; $k_{12}=5.4\times10^6$, $k_{23}=1.2\times10^9$, $k_{34}=2.5\times10^7$, $k_{45}=5.4\times10^9$, $k_{46}=6.4\times10^7$, $k_{67}=1.3\times10^9$, $k_{68}=2.4\times10^7$, $k_{89}=4.0\times10^7$, $k_{90}=6.7\times10^6$.

By using Matlab, the eigenvalues for the two cases are determined, as listed in Table 1 (without work piece case) and Table 2 (with work piece case).

Table 1 Calculated eigenvalues of Case 1 (without workpiece)

Mode number	Natural frequency/Hz	Mode number	Natural frequency/Hz
1	199	6	698
2	254	7	1 548
3	390	8	10 086
4	430	9	18 967
5	609		

Table 2 Calculated eigenvalues of Case 2 (with workpiece)

Mode number	Natural frequency/Hz	Mode number	Natural frequency/Hz
1	199	6	697
2	253	7	1 548
3	390	8	10 086
4	430	9	18 967
5	606		

2.4.2 Forced vibration responses

To solve the forced vibration problems, the number of teeth of gears are given as z_1=25, z_2=84, z_4=23, z_6=84, z_7=24, z_8=141, z_9=24, z_{10}=84, z_{11}=23, z_{12}=84 and z_{13}=25.

When the desired spindle speed input is given, as shown in Fig.5, the associated driving torque for the cases 1 and 2 can be determined, as shown in Fig.6.

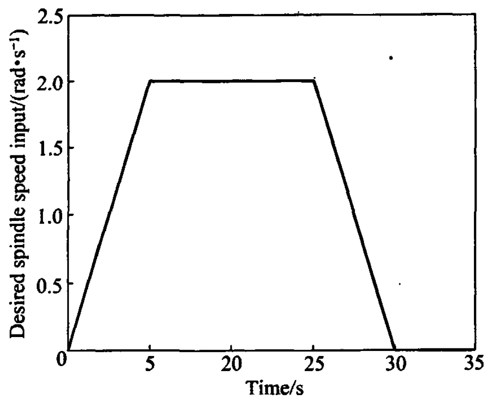

Fig.5 Desired spindle speed input

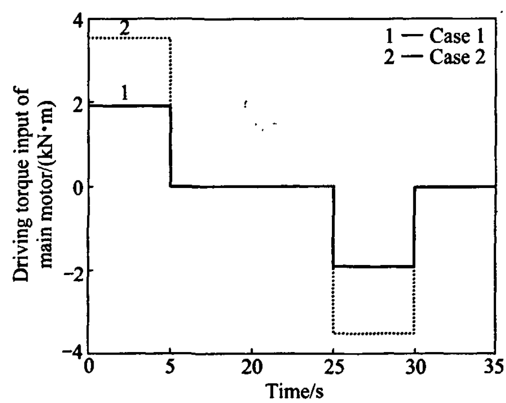

Fig.6 Corresponding driving torque input of main motor in case 1 and case 2

Fig.7 shows the comparison of the angular velocity responses of the spindle end for cases 1 and 2. The pure torsional velocity response of the spindle, ω_φ, is defined as $\omega_\varphi=\omega_7-\omega_d$. And the pure torsional velocity response of the spindle at period of constant spindle speed ω_d=2 rad/s is also computed, as shown in Fig.8. The resultant unbalance torque, $\boldsymbol{T}_u(t)$, is obtained in the case of constant spindle speed ω_d=2 rad/s and shown in Fig.9. Using the same solving method, the pure torsional vibration responses of the spindle are obtained, as shown in Fig.10.

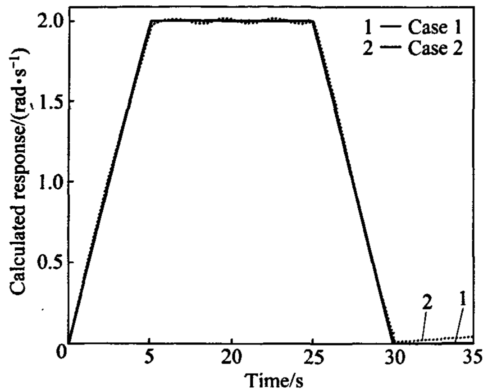

Fig.7 Comparison of calculated angular velocity at face plate of spindle system ω_7 in case 1 and case 2

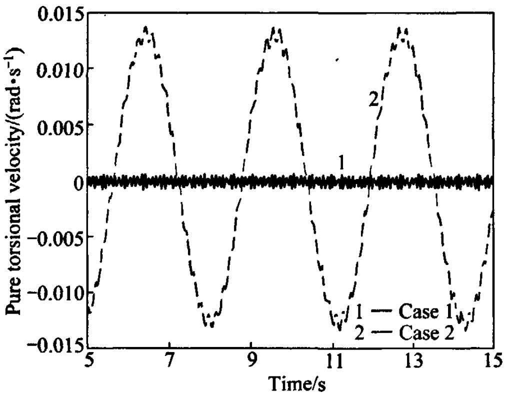

Fig.8 Comparison of pure torsional velocities at period of constant spindle speed ω_d=2 rad/s in case 1 and case 2

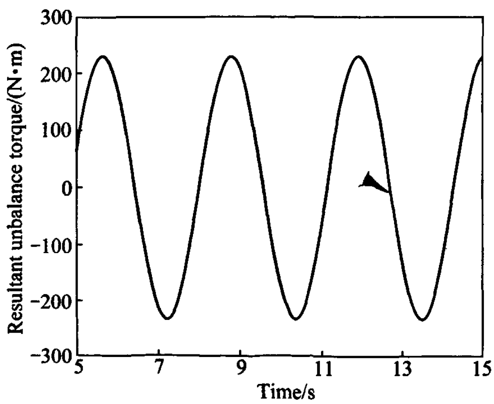

Fig.9 Resultant unbalance torque, $T_u(t)$, at period of constant spindle speed ω_d=2 rad/s

Judging from the computed pure torsional velocity response of the spindle shown in Fig.8, the spindle system cannot operate in the desired constant rotating speed if the gravitational unbalance torque is engaged.

As shown in the Fig.10, the pure torsional velocity response of the spindle increases infinitely near the spindle speed corresponding to the system resonant frequencies. And the pure torsional velocity responses of the spindle with the other spindle speeds are not so small to neglect.

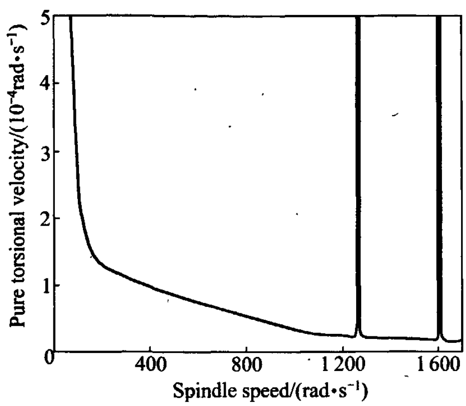

Fig.10 Pure torsional vibration response ω_φ of spindle

3 Conclusions

1) The spindle system of a lathe with unbalanced work piece cannot operate in the desired constant rotating speed as far as the gravitational unbalance torque is engaged.

2) The pure torsional velocity amplitude of the spindle increases infinitely near the spindle speed corresponding to the system resonant frequencies, so the spindle system cannot operate normally with these speeds.

3) The pure torsional vibrations of the spindle cannot be neglected during other spindle speed ranges, so it may cause bad effects on machining accuracy.

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